Concrete pavement design

Portland Cement Association

for roads and streets carrying all classes of traffic

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PORTLAND CEMENT ASSOCIATION

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FOREWORD

THE design of concrete pavements is based upon a vast amount of data which have been compiled during years of research and practical experience. These data from observations of pavements in service, from test roads, laboratory research, field investigations and mathematical computations have removed much of the necessity for guesswork or rule-of-thumb methods in concrete pavement design. The purpose of this manual is to assemble and reduce to workable form as much of these data as are needed by the busy practicing engineer.

Much of the data presented herein was originally discussed in *Concrete Road Design, Simplified and Correlated With Traffic* (1933) by the late Frank T. Sheets, then consulting engineer and later president of the Portland Cement Association. Three editions of that book were prepared and distributed. To make the new manual more complete, sections have been added on the use of steel in concrete pavements and on jointing. Data on traffic volume and on size and frequency of wheel loads characteristic of the several classes of thoroughfares have been revised to conform with information developed by the State-Wide Highway Planning Surveys.

Every effort has been made to present the design data and procedure in such a way as to be most convenient for use by the practicing engineer. The method of design is simple and does not require any profound theorizing. No unproved engineering principles are involved.

Specifically, the design procedure is based on three principles:

1. That the pavement cross-section should be adjusted to the load and subgrade.

2. That pavements should be designed for the controlling wheel loads with a factor of safety of 2, or, in other words, with working stress under such loads limited to one-half the ultimate breaking strength of the concrete.

3. That the fatigue behavior of the pavement slab under heavier loads should be evaluated as a basis for determining the capacity of the pavement to carry these heavier loads.

This issue of the manual is based on the design principles and formulas presented in the previous edition. However, the use of four-element charts has made it possible to simplify the design procedures and reduce the amount of computation required to solve a design problem. The data on traffic have been brought up to date. Recommendations on jointing and the use of subbases have been revised in order to put into practice the knowledge gained from the many experimental projects constructed and the extensive field condition surveys made since the first edition of this manual was published.

It must be emphasized that the stress formulas and charts presented herein are applicable to surface vehicles only. For computing the stresses induced by large airplanes with much greater wheel loads and with tires which have much larger areas of contact with the pavement than are characteristic of street and highway traffic, special design charts developed for airport traffic should be used. For information on design of concrete pavements for airports, write the nearest office of the Portland Cement Association. Upon request literature will be sent free only in United States and Canada.

STRUCTURAL DESIGN of Concrete Pavements

A CONCRETE PAVEMENT slab is a structural unit that can and should be designed for the loads it is to carry. If a slab is to render the years of satisfactory and economical service that can rightfully be expected of it, the designing engineer must take into account four important factors when preparing his design. These are:

(1) Amount, type and weight of present and anticipated traffic;
(2) Supporting power and character of the subgrade;
(3) Climatic region in which pavement is to be constructed; and
(4) Strength and quality of the concrete to be used.

Should one or more of these factors be disregarded, the resulting pavement will be uneconomical. If overdesigned, it will give good service but will have an excessive first cost. If underdesigned, it will be short lived or costly to maintain and it will be both unsatisfactory and uneconomical.

In order that each of the above four factors may be properly evaluated when designing a concrete pavement slab they are discussed in detail in the following paragraphs.

Traffic

The volume and character of the present and anticipated traffic determine the number and width of traffic lanes required. The weight and frequency of the wheel loads of this traffic form the basis of the slab's structural design.

Before undertaking the structural design of a slab it is necessary to determine as accurately as possible the wheel load characteristics of the traffic the pavement will have to carry. Table 1 shows the type of information that should be assembled when preparing traffic data for use in a design problem. It illustrates the range in volume of commercial traffic and in the number of heavy wheel loads that can be expected on the four general classes of highways. It also emphasizes the fact that within each class there are wide variations in traffic characteristics because of the nature of the area served.

The wheel load characteristics of traffic which a specific highway or street will be called upon to carry, after improvement, can be estimated. Data available from the State-Wide Highway Planning Surveys furnish a basis for this estimate. A study of the commercial characteristics of the area in which the highway is located, the population density, the relation of the proposed improvement to other highways and traffic counts on the specific road under consideration, as well as on nearby highways of comparable significance, will enable the highway engineer to make conservative and rational traffic predictions. These predictions may be checked by a study of traffic volumes and wheel loads found on existing roads of similar character in other areas. This is entirely satisfactory, for traffic

Table 1—Typical Traffic Volumes and Wheel-Load Distribution on Four Classes of Highways

Class of highway	Range	Average daily traffic						Less than 4,000 lb.		4,000 to 5,000 lb.		5,000 to 6,000 lb.	
		Total vehicles	Automobiles		Commercial vehicles								
			Number	% of Total vehicles	Number	% of Total vehicles	Number of axles* (wheel loads)	No.	%	No.	%	No.	%
I Primary route in or near a metropolitan area	Avg.	8,179	6,489	79.3	1,690	20.7	4,447	2,889	64.97	246	5.53	239	5.37
	Max.**	13,250	11,236	84.8	2,014	15.2	5,622	3,414		378		347	
	Min.**	3,276	2,686	82.0	590	18.0	1,818	1,350		89		57	
II Primary route in a rural area	Avg.	3,522	2,741	77.8	781	22.2	2,072	1,406	67.85	108	5.21	102	4.92
	Max.	4,128	3,284	79.6	844	20.4	2,228	1,576		172		89	
	Min.	2,677	2,275	85.0	402	15.0	1,312	908		82		72	
III Lightly traveled primary route	Avg.	1,186	924	77.9	262	22.1	597	466	78.05	34	5.69	24	4.02
	Max.	1,566	1,182	75.5	384	24.5	955	667		56		49	
	Min.	769	577	75.0	192	25.0	444	276		35		29	
IV. Secondary or county trunk route	Avg.	899	685	76.2	214	23.8	491	392	79.82	22	4.48	13	2.64
	Max.	1,125	889	79.0	236	21.0	479	407		17		7	
	Min.	700	495	70.7	205	29.3	550	419		27		13	

* Each axle of a truck delivers one wheel load on any one part of the pavement. The front axle usually carries much less load than the rear axle, and is then included in a different weight group. When there are two or more rear axles, each is counted as one wheel load.

studies indicate that roads of similar importance in areas of similar character have essentially the same traffic density and wheel load intensities. A special industry such as a coal mine or factory may affect this normal relationship, but such influences may be evaluated.

Many cities have made comprehensive studies of the traffic pattern within their limits which will supply data needed for design of municipal pavements. Where data are unavailable or need amplification, there are well-established methods for making short-count traffic surveys which will supply the necessary information at low cost and in a short time. Guidance in making surveys may be obtained from the Planning Section of the state highway department in each state. *Traffic Engineering Handbook*, Second Edition, 1950, published by the Institute of Traffic Engineers, New Haven, Conn., includes a section on "Traffic Studies" which will be found helpful.

The traffic figures given in Table 1 may be used to design cross-sections suitable for average roads in each class. However, the best practice is to study

On typical roads in seven midwestern states. Data obtained from 1948 state planning surveys.

Distribution of WHEEL LOADS on commercial vehicles
Number of wheel loads per day and per cent of total commercial wheel loads

6,000 to 7,000 lb.		7,000 to 8,000 lb.		8,000 to 9,000 lb.		9,000 to 10,000 lb.		10,000 to 11,000 lb.		11,000 to 12,000 lb.		12,000 to 13,000 lb.		13,000 to 15,000 lb.		More than 15,000 lb.	
No.	%	No.	%	No.	%	No.	%	No.	%	No.	%	No.	%	No.	%	No.	%
287	6.45	302	6.79	289	6.50	127	2.86	40	0.90	18	0.40	4	0.09	4	0.09	2.4	0,05
452		452		336		168		42		0		10.7		10.1		10.1	
91		93		66		59		13.6									
119	5.74	128	6.18	122	5.89	59	2.85	19	0.91	6.7	0.32	2.0	0.10	0.6	0.03		
74		94		109		82		22		8.9		0		0.7			
66		73		61		45		1.7		3.5							
23	3.85	21	3.51	17	2.84	9	1.50	2.2	0.37	0.9	0.15	0.1	0.02				
59		57		43		19		4.3		1.0		0.4		0.2			
26		30		25		13		4.0		1.3							
16	3.25	24	4.88	16	3.25	5.2	1.05	3.0	0.61	0.1	0.02						
11		18		16		1.4		1.4									
17		37		20		10		6.7									

** Based on total vehicles with distribution as reported for that volume of traffic.

carefully the traffic characteristics surrounding each proposed improvement and design accordingly.

Subgrade

The support that the subgrade gives to a concrete pavement is one of the major factors that must be evaluated and taken into account when designing a slab. In concrete pavement design, the supporting power of a subgrade is measured by the method proposed by the late Dr. H. M. Westergaard.* In this method, subgrades are evaluated according to their k value. The value of k, Westergaard's modulus of subgrade reaction, represents the load in pounds per square inch on a loaded area of the subgrade divided by the deflection in inches

*H. M. Westergaard, "Stresses in Concrete Pavements Computed by Theoretical Analysis" *Public Roads*, April 1926, Vol. 7, No. 2, and "Analytical Tools for Judging Results of Structural Tests of Concrete Pavements", *Public Roads*, December 1933, Vol. 14, No. 10. Dr. Westergaard was, at the time of his death, Gordon McKay Professor of Civil Engineering, Graduate School of Engineering, Harvard University.

of the subgrade under that load. It is expressed in "pounds per square inch per inch (psi/in.)."

Subgrade support may range between rather wide limits, depending upon the type of soil, its density and its moisture content. Soft plastic clay soils may have a modulus of subgrade reaction, k, of approximately 50, while densely graded, nonplastic sand gravels may have k values of 500 or more.

Well-compacted clay soils have a high k value when dry and hard. However, their supporting value decreases considerably with increases in moisture content. Extensive field studies have been made of the moisture content of subgrade soils under concrete pavements. These studies show that under most climatic conditions, clay and clay-type subgrades eventually attain and hold a moisture content approximately equal to their plastic limit. For this reason the k value of a clay or clay-type soil should be determined only when it is at or near its plastic limit. This does not apply to sands or gravels for their supporting power varies little with changes in their moisture content.

In general, subgrade soils should be compacted to maximum density at optimum water content prior to placement of the pavement. Exceptions are high-volume-change soils which, in climates conducive to excessive swelling and shrinkage, often perform best if given maximum compaction at water contents 2 to 4 per cent above optimum. The control of volume change by controlling water content and density is further described on page 42.

It will seldom, if ever, be economical to resort to subgrade treatment or subbases merely for the purpose of increasing the supporting power or k value of a subgrade. The very low unit pressures which are known to exist between a loaded concrete pavement and the subgrade make it unnecessary and uneconomical to build up the subgrade with thick ballast courses in order to increase its bearing capacity. These thick subbases are an essential part of nonrigid pavements where they are needed to reduce unit pressure on the underlying subgrade to a safe point. Except where needed to correct or counteract unsatisfactory subgrade conditions, they have no place in concrete pavement design. Their cost will generally exceed any possible saving in thickness and cost of the concrete slab. (See page 41 for discussion of conditions which do call for subbases under concrete pavements.)

Tables 2 and 3 show the approximate range of values of k for various soil groups as classified by the Highway Research Board, the Bureau of Public Roads, and the Corps of Engineers, Department of the Army. Tne values given are for moisture contents and densities approximating those usually found in subgrades under service conditions. In the absence of measured values of k, the values in these tables may be safely used if the engineer is able to classify the soil and to judge where it falls within the limits of the determined classification. On large projects or where the quality of the subgrade is questionable, or the wheel loads critically high, field tests should be made to determine the actual k values.

The k value of a soil may be determined by loading a rigid plate placed directly on the subgrade, or by loading an existing pavement slab that rests on a similar soil. A detailed description of the test procedures and equipment

Table 2—Approximate Range of *k*-values for Soil Groups of the Highway Research Board Classifications* and of the Bureau of Public Roads Classifications

Major divisions	Soil groups and typical description	Highway Research Board	Original Bureau of Public Roads symbol	Approximate range of *k*-values for each soil group
Gravelly and sandy soils	Well-graded gravel-sand-clay. Excellent binder.	A - 1 - a	A - 1	400-700 or greater
	Sand-clay mixtures. Excellent binder.	A - 1 - b	A - 1	250-575
	Gravel with fines, very silty gravel, poorly graded gravel-sand-clay, and sand-clay mixtures. Poor binder. Friable.	A - 2 - 4 A - 2 - 5	A - 2 friable	300-700 or greater
	Poorly graded clayey gravels, gravel-sand-clay, and sand-clay mixtures. Inferior binder. Plastic.	A - 2 - 6 A - 2 - 7	A - 2 plastic	175-325
	Well-graded gravel, gravel-sand mixtures, and sands. Little or no fines.	A - 1 - a	A - 3	325-700 or greater
	Poorly graded gravel, gravel-sand mixtures, and sands. Little or no fines.	A - 1 - b A - 3	A - 3	200-325
Fine-grained soils in which silt sizes predominate	Predominantly silt soils with moderate to small amounts of coarse material and small amounts of plastic clay.	A - 4	A - 4	100-300
	Poorly graded silty soils which contain mica and diatoms and which have elastic properties.	A - 5	A - 5	50-175**
Very fine-grained inorganic and organic soils in which the clay fraction governs	Clay soils with moderate to negligible amounts of coarse materials. Includes well-graded inorganic silt-clay, sand-silt-clay, and sand-clay soils.	A - 6 A - 7 - 6	A - 6	50-225
	Elastic clay soils with moderate to negligible amounts of coarse materials. Usually poorly graded or contains organic or other materials which make them elastic.	A - 7 - 5	A - 7	50-225

* As given in the Soil Committee Report, Highway Research Board Proceedings, 1945, Vol. 25, page 377, and adopted by the American Association of State Highway Officials.

** Some soils of volcanic origin may have *k*-values greater than those shown for this group.

used in making plate bearing tests to determine a soils k value is given in Appendix I.

If time and equipment are not available to conduct the plate bearing or full-

Table 3—Approximate Range of *k*-values for Soil Groups of the Casagrande Soils Classification as Used by Corps of Engineers, Department of the Army

Major divisions	Soil groups and typical description	Subgrade group symbols	Approximate range of *k*-values for each soil group
Gravel and gravelly soils	Well-graded gravel and gravel-sand mixtures. Little or no fines.	GW	500-700 or greater
	Well-graded gravel-sand-clay mixtures. Excellent binder.	GC	400-700 or greater
	Poorly graded gravel and gravel-sand mixtures. Little or no fines.	GP	300-500
	Gravel with fines, very silty gravel, clayey gravel, poorly graded gravel-sand-clay mixtures.	GF	250-500
Sands and sandy soils	Well-graded sands and gravelly sands. Little or no fines.	SW	250-575
	Well-graded sand-clay mixtures. Excellent binder.	SC	250-575
	Poorly graded sands. Little or no fines.	SP	200-325
	Sand with fines, very silty sands, clayey sands, poorly graded sand-clay mixtures.	SF	175-325
Fine-grained soils having low to medium compressibility	Silts (inorganic) and very fine sands, mo, rock flour, silty or clayey fine sands with slight plasticity.	ML	150-300
	Clays (inorganic) of low to medium plasticity, sandy clays, silty clays, lean clays.	CL	125-225
	Organic silts and organic silt-clays of low plasticity.	OL	100-175
Fine-grained soils having high compressibility	Micaceous or diatomaceous fine sandy and silty soils, elastic silts.	MH	50-175
	Clays (inorganic) of high plasticity, fat clays.	CH	50-150
	Organic clays of medium to high plasticity.	OH	50-125

LEGEND for group symbols

G—gravel.
S —sand.
M—mo, very fine sand, silt, rock flour.
C—clay.
F —fines, material smaller than 0.1-mm. diameter.
O —organic.
W—well-graded.
P —poorly graded.
L —low to medium compressibility.
H —high compressibility.

scale loading tests, an approximate k may be determined by making field or laboratory tests to find a subgrade soil's California Bearing Ratio (CBR) and then converting to k by using the relationships shown in Fig. 1.*

*Based on data presented in "Soil Tests for Design of Runway Pavements", T. A. Middlebrooks and G. E. Bertram, *Proceedings*, Highway Research Board, 1942, Vol. 22, page 144.

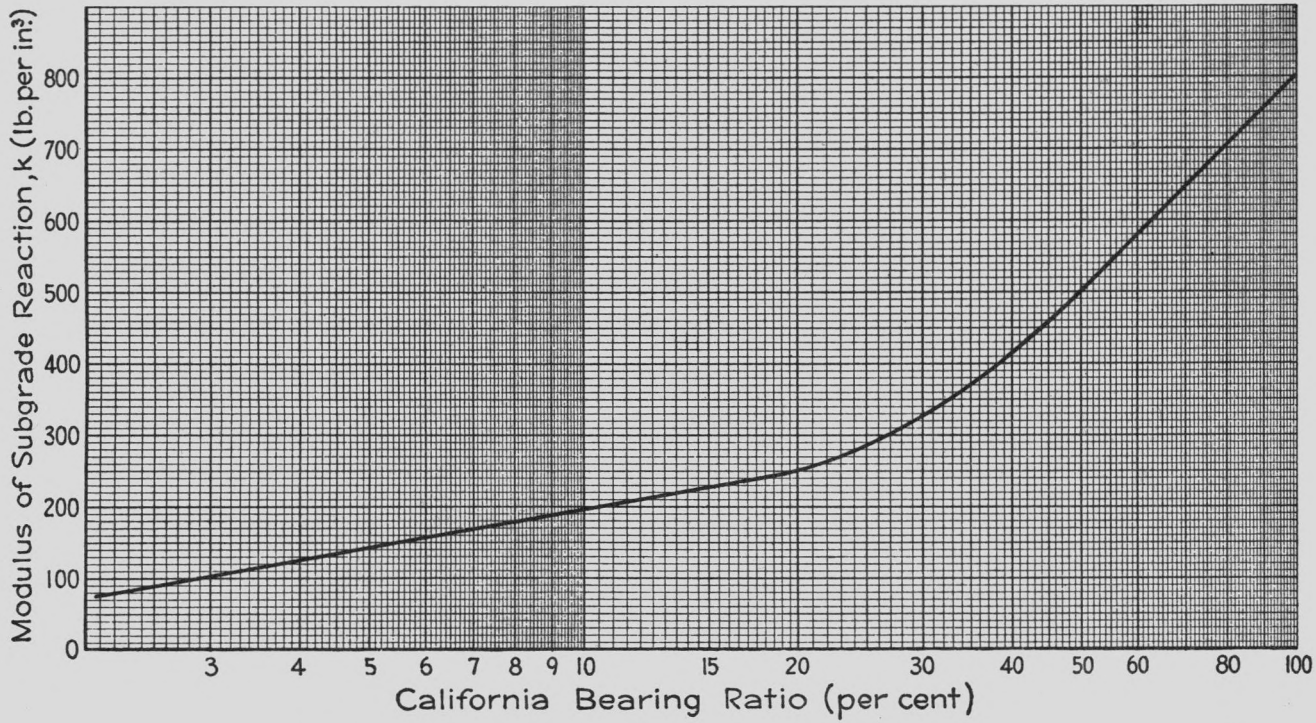

Fig. 1—Relationship between California Bearing Ratio and modulus of subgrade reactions.

Although actual measurements are preferred when previous bearing test data on the soil are not available, it is possible to establish the value of *k*, with a fairly satisfactory degree of accuracy, by correlation with other soils of known *k* value. To illustrate, if the soil physical test constants show that a subgrade is composed of material similar to the better soils falling in the A-4 group (which, from Table 2, have *k* values ranging from 100 to 300), and if previous bearing test data are available showing that soils of similar physical test constants have a *k* value of 200 to 250, then the *k* value of this particular subgrade can, with reasonable accuracy, be estimated at 200 to 250 depending on whether its physical test constants indicate it is slightly better or worse than soil with which it is being compared.

The effect of *k* on pavement thickness is such that a high degree of accuracy is not required in its determination. Small errors in determining or selecting the values of *k* will not seriously affect pavement design. This is illustrated by Fig. 2 which shows graphically the influence of *k* upon pavement thickness.

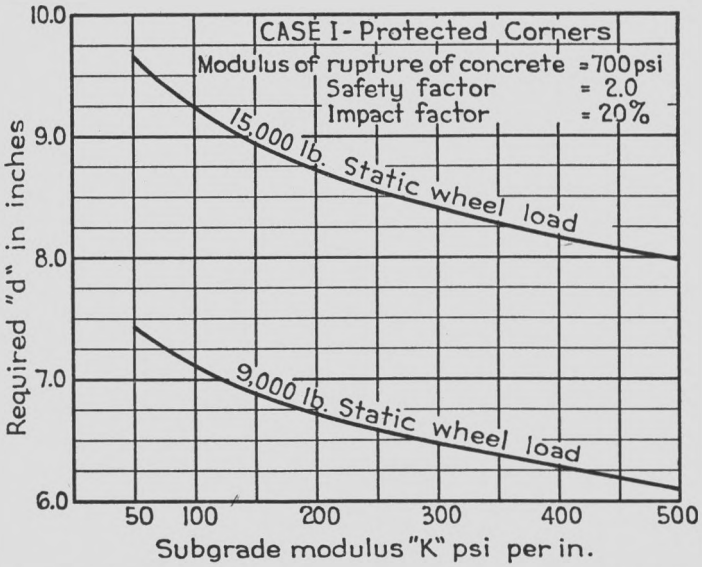

Fig. 2—Influence of subgrade support on required pavement thickness.

Climate

The climatic region in which a project is located determines the type and degree of exposure the pavement will receive. Knowing these, established procedures are followed to design a concrete mix that will have the required durability as well as strength. In northern and north central states where snow, sleet and ice are common and where the maintenance forces use de-icing salts, air-

entrained concrete should be used. In the southern states with their milder climates where icing of pavements does not occur, air-entrained concrete is not required. However, because of its other desirable features, such as increased workability, cohesiveness, less tendency to segregate and improved durability, its use in these areas may be advantageous.

The climatic region also tends to indicate the type of unstable soil conditions that may be encountered on a project. Soils that are satisfactory subgrades under one set of climatic conditions may become unstable or troublesome under other combinations of temperature, humidity and rainfall. A knowledge of the climate and soil type is used when determining the need for type and thickness of subbase or subgrade treatment used under a pavement.

Concrete

When designing concrete pavements the flexural strength (modulus of rupture)* of the concrete is used rather than its compressive strength. This is done for the reason that when a concrete pavement slab is loaded to the point that a break develops, the concrete fails in flexure rather than in compression.

Studies should be made to determine the most economical strength of concrete to use under the existing conditions. The relative cost of cement, aggregate, labor and equipment will determine whether a concrete having a modulus of rupture of, say 800 psi would be more economical than a concrete having a modulus of rupture of perhaps 600 psi. Under average conditions, concrete having a modulus of rupture between 600 and 750 psi at 28 days will be found the most economical.

The selection of a concrete mixture must also be governed by requirements for durability under the climatic conditions to which it will be exposed. The design of paving mixes that will have the desired characteristics before and after hardening is covered in detail in Portland Cement Association manual, *Design and Control of Concrete Mixtures***. The design procedures set forth in this manual should be followed to produce concrete that will have the necessary workability, strength and durability.

Stresses in Concrete Pavement

Stresses in a concrete pavement slab come from two principal sources: The traffic using the pavement and the volume changes that take place within the concrete during and after the hardening period. The wheel loads induce flexural or load stresses. The volume changes that take place in concrete, whether caused by shrinkage during hardening or by changes in the temperature and moisture content of the concrete, produce tensile, compressive and flexural stresses. Before undertaking the structural design of a concrete pavement slab, a designer should have an understanding and appreciation of these stresses.

**Modulus of rupture* is an empirical value representing the stress in the outer fibers of a beam at failure, computed on the assumption that the stress varies directly with the distance from the center (neutral axis) of the beam.

**Available free from Portland Cement Association in U.S. and Canada.

Stresses Due to Loads—The most severe stresses set up in a concrete pavement are flexural stresses caused by wheel loads. Comprehensive studies* have been made of these stresses in concrete pavements under actual and simulated wheel loadings. These studies and observations of pavements in service show that the critical point in a uniform-thickness pavement slab is the corner. Stresses due to a load on the edge of a slab are less than those caused by the same load on a corner of the same thickness. Stresses due to a load in the interior of a uniform-thickness slab are considerably less than those caused by the same load at either an edge or a corner.

A concrete pavement fails under load when the load or bending moment is of such magnitude that the flexural stress reaches the modulus of rupture of the concrete. Formulas have been developed for the determination of the flexural stresses in concrete pavements. Stresses computed by these formulas (see page 17) agree closely with the stresses measured on actual slabs under similar conditions of load, pavement thickness and subgrade support.

Stresses Due to Changes in Temperature and Moisture—When the top of a pavement is cooler than the bottom, the top is in tension and the bottom is in compression. When the top is warmer than the bottom, the condition is reversed, placing the top in compression and the bottom in tension. These temperature differentials within the slab tend to curl the slab edges downward in the daytime and upward at night. This tendency to curl, resisted by the weight of the slab, results in curling** stresses. Differences in moisture content between top and bottom of the slab have a similar effect.

The significance of slab curling is not the stresses produced during curling, but rather that the slab corners or edges tend to lift away from the subgrade. This reduces the support given a pavement by the subgrade and increases the flexural stresses induced by the wheel loads.

At slab corners where load stresses are the greatest, the curling stresses are negligible for the reason that the tendency of the slab to curl is resisted by a very small amount of concrete. The previously mentioned formulas for the determination of the flexural stresses at a corner were developed so as to give results which agree with actual stress measurements at a corner, when the corner is curled upward. Therefore, the influence of curling on corner stresses is amply taken care of by the use of the formulas or by use of the charts developed from them.

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*Teller and Sutherland, "The Structural Design of Concrete Pavements", Part 4, *Public Roads* Sept.-Oct. 1936, Vol. 17, Nos. 7 and 8.

**_Curling_—"the distortion of the pavement slab from its proper plane caused by differential expansion or contraction resulting from a difference in moisture content or temperature between the top and bottom of the slab."

Warping—"the distortion or displacement of the pavement from its proper plane caused by external forces" (other than loads).

Definitions are taken from "Report of Committee on Warping of Concrete Pavements", *Proceedings*, Highway Research Board, 1945, Vol. 25, page 203.

In the interior of the slab or along the edge at some distance from the corner, curling stresses frequently do exist which, under some circumstances, are additive to load stresses. However, as previously explained, stresses due to loads at edges and interiors of uniform-thickness slabs are less than those caused by the same load on a corner. This is also true of thickened-edge slabs designed as hereinafter described. The margin of strength present in the interior and edges of a slab is sufficient to offset the curling stresses which may occur there.

Curling stresses are not always additive to load stresses but are frequently subtractive, so that the resultant stress is the load stress *less* the curling stress. In fact, this latter condition prevails for a much greater portion of the time than does the reverse.

One reason is that the bottom of a slab, lying on a moist subgrade, normally has a higher moisture content than the top. It is therefore in a more expanded condition because of moisture and tends to cause curl in a direction opposite to that caused by higher top temperature. To offset this greater moisture content in the bottom, the top of the slab must be appreciably warmer than the bottom in order to establish a condition of zero curling. Until the effect of temperature differential between the bottom and top is greater than that of the moisture differential, the slab will not be cupped downward, which is the condition of the interior or edge portions when load and curling stresses are additive.

Such long-time temperature records as are available indicate that the temperature at the bottom of the slab exceeds that of the top more than the reverse. Since the temperature of the top would have to exceed that of the bottom by a considerable margin (depending on the difference in moisture content) before curling stresses and load stresses become additive, it will be understood why the statement is made that curling stresses in the interior and edge are subtractive more frequently than they are additive.

Maximum stresses produced by additive curling and load stresses will occur but infrequently. With the reserve strength which is present in the edges and interiors of slabs, the maximum stress in comparatively short slabs will seldom exceed the working stress used in design of the slab corners. This is confirmed by the performance of pavements in service. In the infrequent cases where the assumed working stress may be exceeded, the factor of safety will be sufficient to prevent rupture, and because of the infrequent occurrence of the high stress, the effect of fatigue, as discussed later, will be of little importance.

For reasons just outlined it is unnecessary, under normal conditions and with the jointing arrangements recommended herein, to reduce the allowable working stress due to loads in order to compensate for curling stresses.

Concrete pavement designed by the formulas and charts presented herein and properly jointed as described later, have excess strength to offset these curling stresses.

Seasonal changes in the temperature and moisture content of a concrete pavement produce measurable changes in the volume (length) of a slab. These seasonal changes in volume produce compression and tension in a slab. These

stresses are kept within safe limits by the proper jointing of the pavement*, and need not be taken into account when computing pavement stresses by the method described later.

Stress Computation—To produce an economical pavement design, for a known combination of traffic and subgrade conditions, it is necessary to determine the stresses that will be set up in that pavement by the moving wheel loads. The practicability of doing this can no longer be questioned since investigators have been able to show close agreement between calculated and measured stresses.

Several methods of computing stresses have been proposed. The best known of these is the mathematical analysis developed by Dr. H. M. Westergaard**. This analysis has been subject to comprehensive studies at the Arlington Laboratory of the Bureau of Public Roads where stresses were measured in pavements under load.

When a corner of a pavement slab is curled upward because of differentials in moisture content and temperature between the top and bottom of the slab, the support of the subgrade is reduced. Stresses in the concrete caused by a load on the slab corner when it is in this curled upward position will be higher than indicated by Westergaard's corner formula. Also, stresses in the slab are not distributed exactly as assumed in the derivation of the Westergaard formula. Several modifications of this formula have been proposed to take these factors into account.

Dr. Gerald Pickett*** has prepared a discussion of Westergaard's formula and three of the better known modifications that have been proposed. This discussion, which also gives the derivation of the formulas he developed for the determination of flexural stresses in a concrete pavement slab, is contained in Appendix III.

In the development of his formulas, Dr. Pickett provided for the condition existing when a slab corner is curled upward by assuming that subgrade support was lacking for some distance from the corner. These formulas are semi-empirical. They give results in close agreement with observed stresses in the Arlington study. When the computations from the basic corner formula are plotted, as in curve 8 of Fig. A of Appendix III, the resulting curve agrees closely over a considerable range with theoretical curves which are based on reasonable assumptions as to subgrade support and distribution of stresses. These semi-empirical formulas are suitable for the full range of conditions encountered in the design of modern street and highway pavements.

Provision may or may not be made for load transference across the opening at corners formed by the intersection of transverse cracks or joints with the free

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*A. A. Anderson, "Expansion Joint Practice in Highway Construction", American Society of Civil Engineers *Proceedings*, 1948, Vol. 72, page 1083.

**See footnote, page 7.

***Formerly research physicist, Research Laboratories of the Portland Cement Association, now Professor of Mechanics, University of Wisconsin.

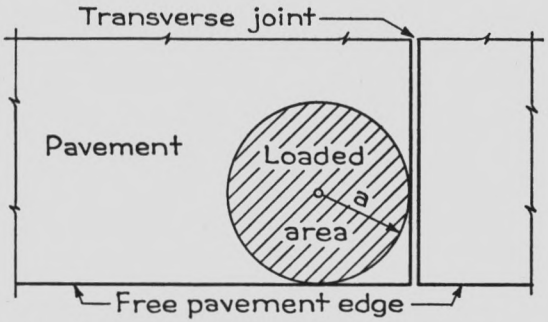


Fig. 3—Position of wheel load that produces greatest flexural stress in a pavement.

edge of a pavement (see Fig. 3). If provision has been made to transfer at least 20 per cent of the load from one slab corner to the other, by some adequate mechanical device or by aggregate interlock, these corners are said to be "protected". Where there is no adequate provision for load transference and one corner must carry over 80 per cent of the load, the corners at such a location are said to be "unprotected". Dr. Pickett has developed a separate formula for the determination of the flexural stresses in a concrete pavement for each of these cases.

For convenience these semi-empirical formulas are presented as follows:

Case I—Protected Corners: $$S=\frac{3.36P}{d^2}\left[1-\frac{\sqrt{\frac{a}{l}}}{0.925+0.22\frac{a}{l}}\right] \quad \text{(Formula 1)}$$

Case II—Unprotected Corners: $$S=\frac{4.2P}{d^2}\left[1-\frac{\sqrt{\frac{a}{l}}}{0.925+0.22\frac{a}{l}}\right] \quad \text{(Formula 2)}$$

in which

S = maximum tensile stress in psi at the top of the slab in a direction parallel to the bisector of the corner angle, due to a wheel of P lb.

P = wheel load in lb. placed on the slab corner in the position indicated in Fig. 3. P is the static wheel load increased by a factor to provide adequate allowance for the impact of moving loads.

d = thickness in in. of a uniform-thickness concrete slab at a corner (or equivalent thickness of a thickened-edge slab).

a = radius in in. of the circular area equivalent to the contact of the tire with pavement.

l = radius of relative stiffness defined by the equation

$$l=\sqrt[4]{\frac{Ed^3}{12(1-\mu^2)k}} \quad \text{(Formula 3)}$$

(See Table 11, page 72, for values of l.)

E = modulus of elasticity of the concrete

μ = Poisson's ratio for the concrete

k = modulus of subgrade reaction in psi/in.

Formulas 1 and 2 as presented permit the pavement designer to vary basic factors as may be desirable under special conditions, and to take into account unusual highway wheel loadings and correspondingly larger tire contact areas.

Though the foregoing formulas (1 and 2) are suitable for the full range of conditions encountered in the design of streets and highway pavements, they are not applicable for computing the stresses induced by large airplanes with their much greater wheel loads and larger tire contact areas. For airport pavement design, the special design procedures given in Portland Cement Association manual *Design of Concrete Airport Pavement** should be used.

The flexural stress (S) for any combination of circumstances (wheel load, subgrade support and concrete strength) may be readily determined by formulas 1 and 2. However, it is not practical to solve directly for d, the thickness of pavement required, because the value of d influences the value of l. For convenience in determining the values of d or S, charts based on formulas 1 and 2 have been prepared. These design charts (Figs. 4, 5, 6 and 7) permit a direct solution of the formulas for gross wheel loads (P) of 3000 to 24,000 lb. and for modulus of subgrade reaction (k) ranging from 50 to 500 psi/in.

Constants Used in Charts—In developing the charts the following have been used for the various factors:

P = the gross wheel load including an allowance for impact. An impact allowance of 20 per cent is common for wheel loads on pneumatic tires. Thus, if the static wheel load is 10,000 lb. and if 20 per cent is to be allowed for impact, P would be $1.2 \times 10{,}000 = 12{,}000$ lb.

d = the thickness of a uniform-depth slab or its equivalent for a thickened-edge slab. (The latter is calculated as explained on pages 23 to 25.)

$E = 4{,}000{,}000$ psi

$\mu = 0.15$

a = the radius of an area, which was calculated separately for each load, using data supplied by major tire manufacturers and by the Tire and Rim Association. For single tires a was taken as the radius of a circle having an area equal to the average area of contact reported by the tire manufacturers. For dual tires a was taken as the radius of a circle having an area equal to the sum of the area of contact of the two tires plus the area between them, the imprint of each tire being assumed to be a circle. Fig. 8 gives value of a for varying loads.

Case I—Protected Corners—Protected corners are those at which provision is made for transferring at least 20 per cent of the load across the intervening

*Available free from Portland Cement Association in U. S. and Canada.

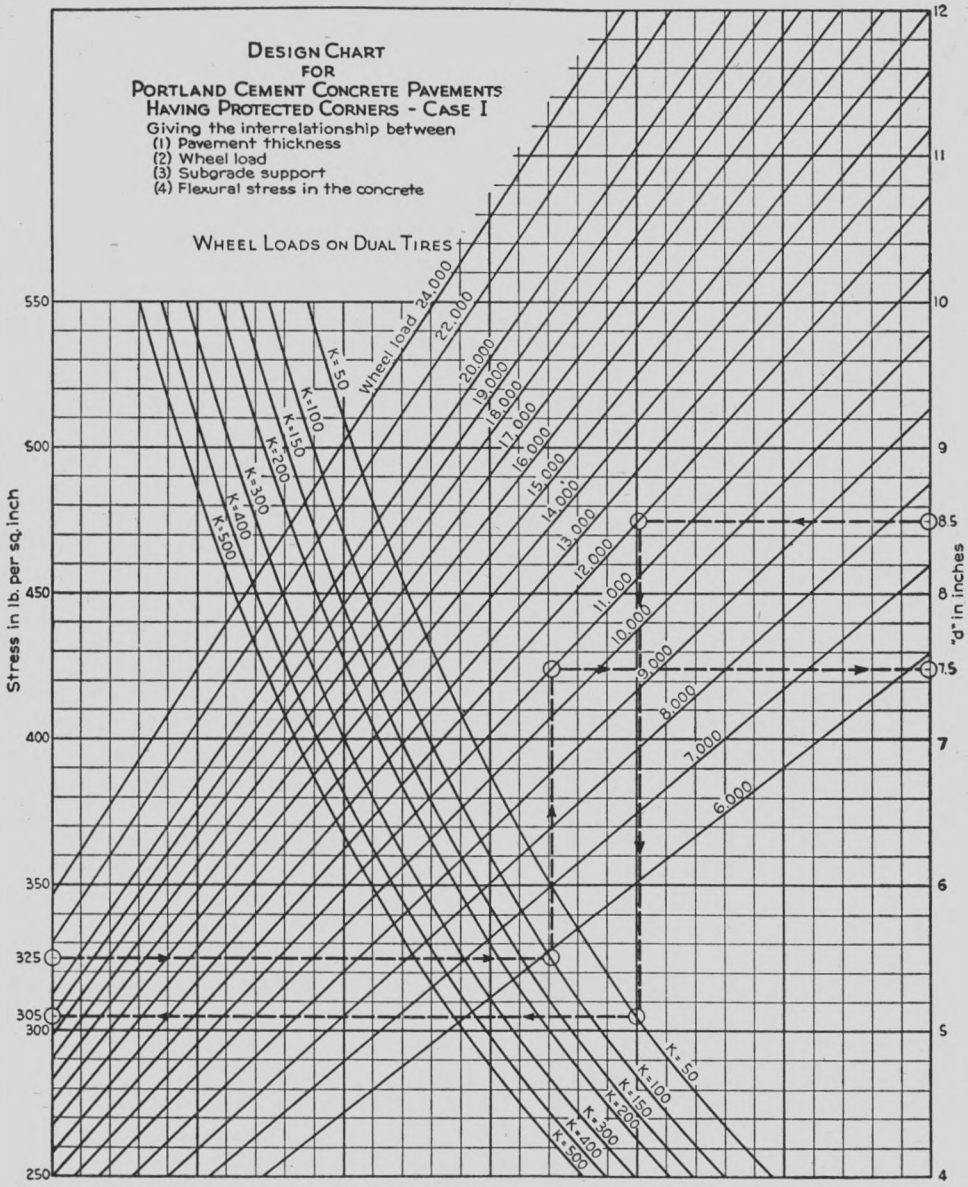


Fig. 4

joint so that they aid one another in carrying loads. Flexural stresses determined by use of Formula 1 or read direct from Figs. 4 and 6 are for pavements with protected corners. This formula and its corresponding design charts may be used when 20 per cent or more of the gross wheel load is transferred from the loaded corner to the adjacent slab. There are several recognized methods of providing this amount of load transference.

1. Aggregate interlock. At natural cracks or at cracks which form beneath the groove or ribbon of transverse dummy-type joints (see Fig. 23) the

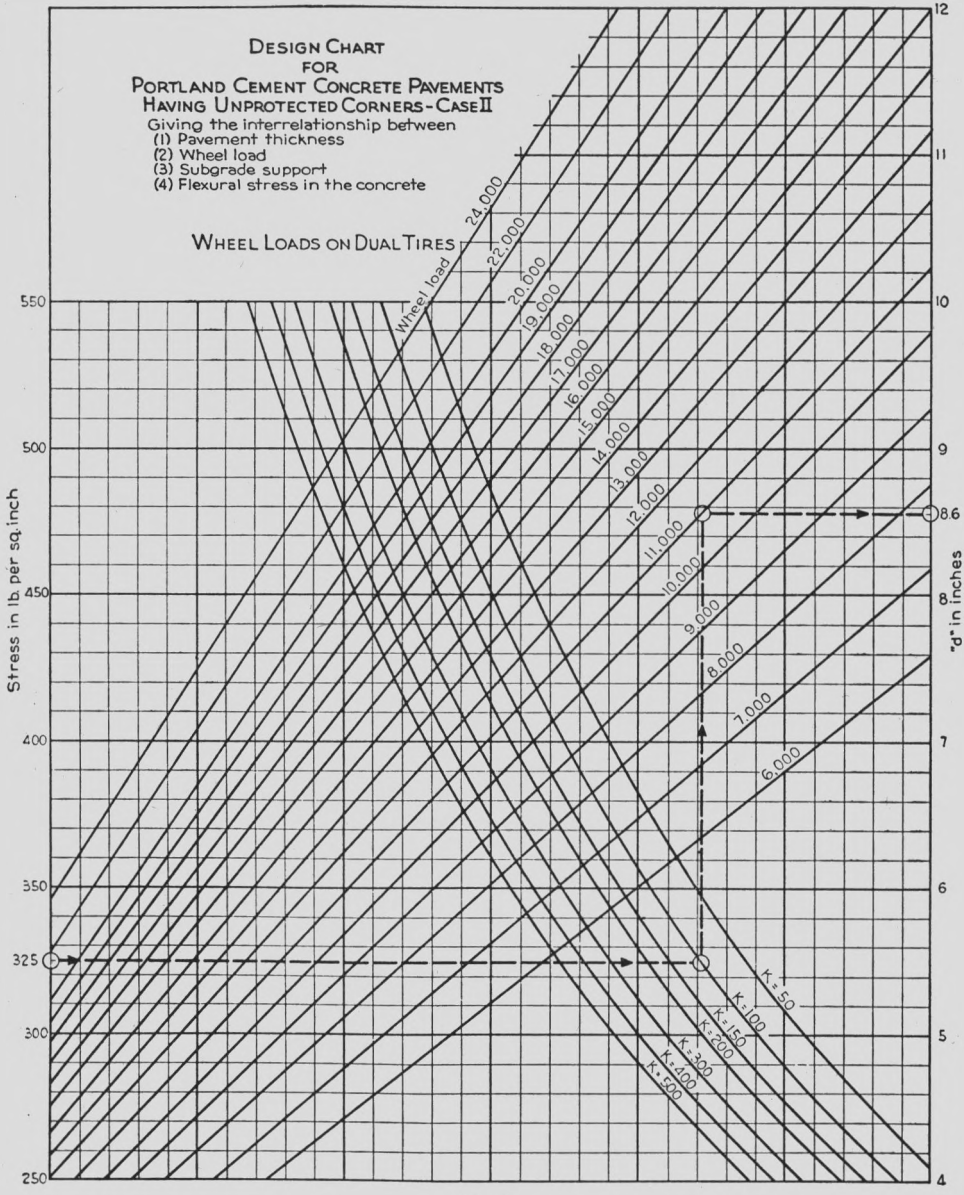


Fig. 5

rough faces of the crack mesh sufficiently to prevent slab end deflection, providing these cracks do not open too wide. A detailed discussion of this subject is given on page 65.

2. Slip dowels*. These usually are smooth, round steel bars ¾ to 1 in. in diameter, 13 to 20 in. long and spaced 8 to 18 in. apart. Square bars, steel pipe and small channels are sometimes used. Several proprietary

*The term *dowels* as used in this text includes all adequate mechanical load-transfer devices.

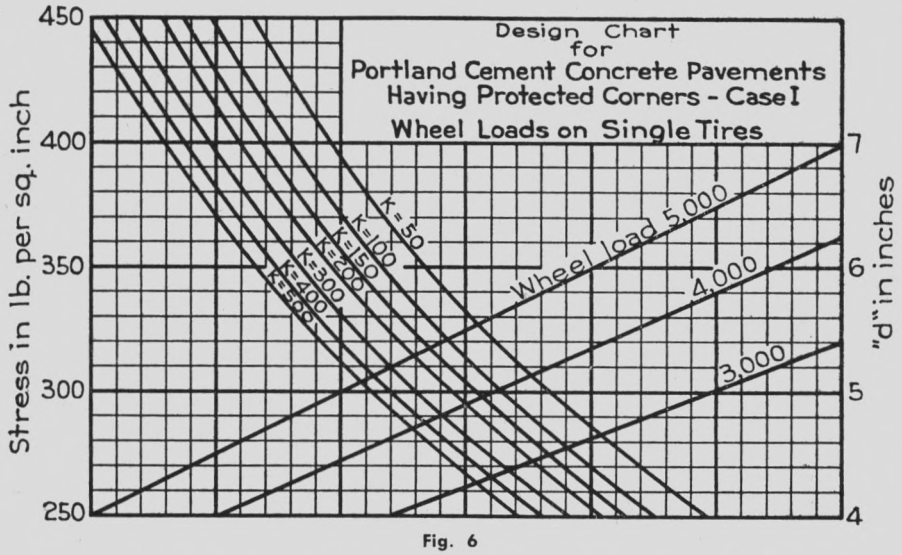
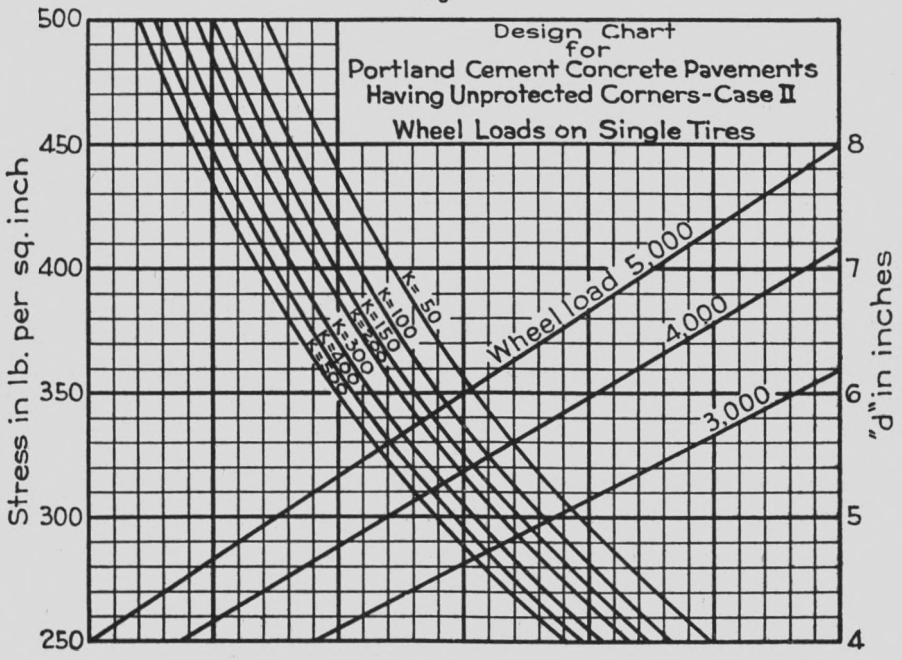


Fig. 6

load-transfer devices also come under this classification. A detailed discussion of design and installation of dowels is given on page 54.

3. Sills. A metal support, embedded in one slab and extending under the bottom edge of the adjacent slab, is sometimes used. A small concrete slab set at subgrade elevation on which the ends of the slabs rest has been used. While not strictly a load-transference device, this concrete sill serves the same purpose.

Fig. 7



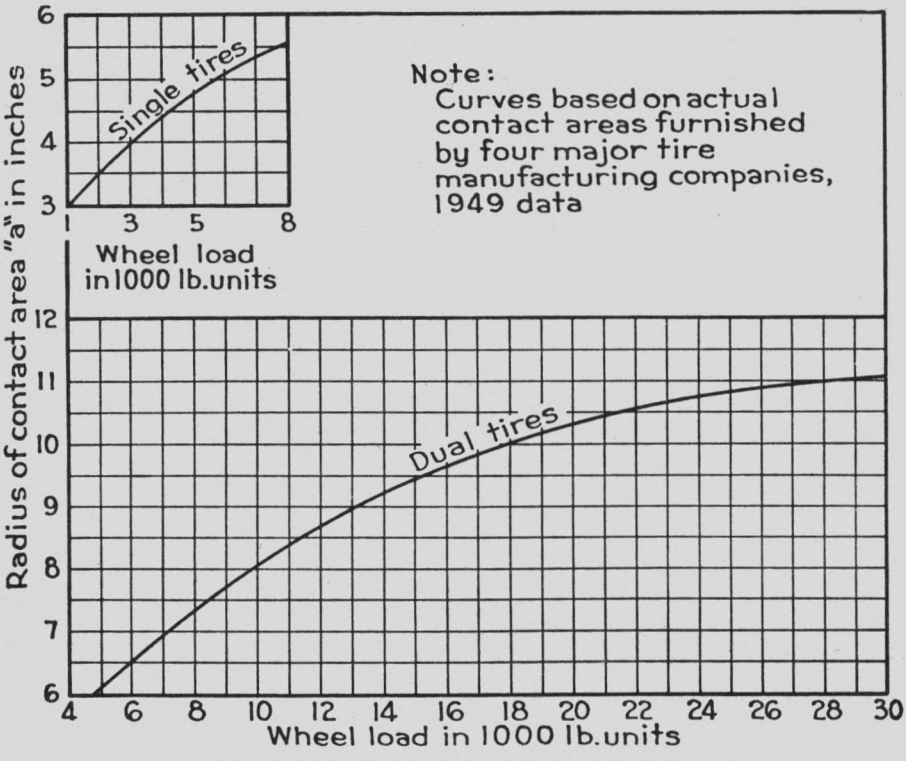

Radius of contact area "a" in inches
Single tires
Wheel load in 1000 lb. units
Note:
Curves based on actual contact areas furnished by four major tire manufacturing companies, 1949 data
Dual tires
Wheel load in 1000 lb. units

Fig. 8—Radius of contact area a for various wheel loads.

Where pavements designed under Case I are to have thickened-edge cross-sections (see Fig. 9), load transference must be provided at all longitudinal joints in the thin interiors of the slab. Adequate load transfer is assured at such longitudinal joints if the dummy-type or the keyed-type joint (see Fig. 22) are used providing the faces of the joint are held tightly together by tiebars. This is necessary in order to insure the structural integrity of the thin portions of such slabs. The design of tiebars is discussed on page 49.

Case II—Unprotected Corners—Unprotected corners are those at which there is no adequate provision for load transference. This condition exists at undoweled expansion joints or at cracks or undoweled contraction joints which do not meet the requirements outlined for Case I. Flexural stresses determined by use of Formula 2 or read direct from Figs. 5 and 7 are for pavement with unprotected corners.

Pavement Cross-Sections

The principal types of cross-sections used for concrete pavements are illustrated in Fig. 9. As previously pointed out (page 14), the flexural stresses induced in a pavement by a load in the interior of a uniform-thickness slab are considerably less than those caused by the same load located at either an edge or a

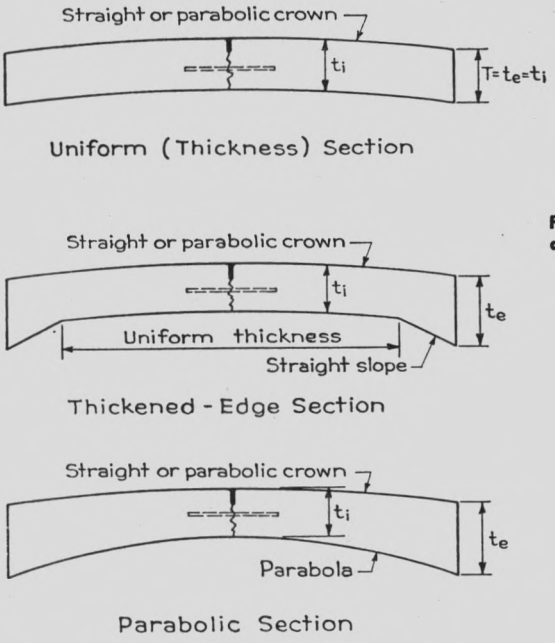


Fig. 9—Principal types of cross sections.

corner. The differences in edge, corner and interior stresses under the same load are such that the thickness of the interior portion of a slab may be reduced materially without having the stresses exceed those in the edges and corners. The thickened-edge and parabolic sections shown in Fig. 9 were developed to take advantage of this fact. They make more efficient use of materials than does a uniform-thickness slab of equal load-carrying capacity. Obviously the most economical cross-section is one in which the edge and interior thickness of the pavement are such that the stresses at those positions are equal under a given load. Such a cross-section is said to have a balanced design.

Design of Balanced Cross-Sections—If it is desired to take advantage of the economies of a balanced cross-section, the slab should be designed so that both interior and exterior portions have equal strengths. The exterior corners of such a slab must have flexural strengths equal to that of the critical portion of a uniform section whose thickness is equal to *d* obtained by solving Formula 1 or read from Figs. 4 and 6. The following empirical methods* of obtaining the equivalent *d* of a thickened-edge section give results so close to those computed by the Westergaard** method that resulting stresses agree within 2 or 3 psi.

By this empirical method the equivalent value of *d* for most thickened-edge sections is the average thickness of the slab along the base *AB* (see Fig. 10) of

*Developed by the late Frank T. Sheets, former president of the Portland Cement Association and first presented by him in 1933.

**Dr. H. M. Westergaard, unpublished analysis, see footnote, page 7.

an isosceles triangle which has the corner of the slab as a vertex and an altitude of 24 in. This is the same as the average thickness along the line AC which can readily be calculated for any edge thickness, t_e, center thickness, t_i, and edge slope. This applies to all parabolic sections, and to all sections in which the edge thickening is obtained by a straight slope extending from the edge a distance ranging from 2 to 4 ft.

For cross-sections having a uniform thickness extending 1 or 2 ft. from the edge and then sloping in a distance of 2 ft. or more to the center thickness, d is the average thickness along the base of the triangle when the altitude is 30 in.

The foregoing method of obtaining the equivalent value of d for a thickened-edge slab applies to all of the various thickened-edge sections. For a balanced design, one in which the stresses in interior and edge are approximately equal and all parts of the slab are working at the same efficiency, experience and research have shown that the center thickness should be ⅔ of the edge thickness when the increase in thickness is obtained by a straight slope in the outer 2 ft. of the slab. Center thickness greater than this is unnecessary and uneconomical because the interior of the slab would then have greater strength than the edges and corners.

Extensive traffic tests and road service records have shown that for sections so proportioned the corner rather than the interior or unbroken edge of the slab is the critical section. Therefore, if this ratio is adhered to for determining center thickness, corner stress analysis is sufficient.

Table 4—Relation of d, t_e and t_i for Balanced Cross-Sections

For all cross-sections, interior thickness $t_i = 0.85d$

Shape of edge thickening	Required ratio $\frac{t_e}{d}$	Required ratio $\frac{t_e}{t_i}$
Straight slope of		
2 ft. 0 in.	1.275	1.500
2 ft. 3 in.	1.227	1.444
2 ft. 6 in.	1.189	1.399
2 ft. 9 in.	1.159	1.364
3 ft. 0 in.	1.134	1.334
3 ft. 3 in.	1.116	1.313
3 ft. 6 in.	1.102	1.296
3 ft. 9 in.	1.091	1.284
4 ft. 0 in.	1.082	1.273
1-ft. uniform from edge then 2-ft. slope	1.115	1.312
1-ft. 6-in. uniform from edge then 2-ft. slope	1.062	1.250
2-ft. uniform from edge then 2-ft. slope	1.030	1.212
Parabolic, width		
9 ft.	1.148	1.351
10 ft.	1.127	1.326
11 ft.	1.111	1.307
12 ft. 6 in.	1.094	1.287
18 ft.	1.059	1.246
20 ft.	1.052	1.237
22 ft.	1.046	1.231
24 ft.	1.042	1.225
25 ft.	1.040	1.223

When the edge thickening is obtained by using a 2-ft. slope and the interior thickness is ⅔ of the edge thickness, as shown in Fig. 10, the section is balanced and the following relationships exist:

Edge thickness, $t_e = 1.275\ d$ (Formula 8)

and

Interior thickness, $t_i = 0.85\ d$ (Formula 9)

Since it is obvious that the interior stresses are not influenced by the manner in which the edge thickness is obtained, Formula 9 will hold regardless of the shape of the edge thickening.

If the shape of the edge thickening differs from that shown in Fig. 10, the edge thickness, t_e, must be of such dimensions that the equivalent value of d, determined as previously described, is at least equal to the required value of d as determined for Case I. In other words, the edge thickening may be varied at will so long as the resulting value of d is not less than that required.

Frequently it may be convenient to use an edge thickness other than $1.275d$. A careful selection of cross-section may permit available side forms to be used without unbalancing the design or modifying the required value of d. Table 4 presents, for various shapes of edge thickening, the ratios of $\frac{t_e}{d}$ and $\frac{t_e}{t_i}$ which will result in balanced designs.

It is not essential that the center thickness be in even inches or half-inches, since subgrading equipment can be adjusted for any depth. However, for practical reasons it is usually taken at even quarter-inches, increasing the center thickness above calculated values to the next even quarter-inch where necessary.

After a center thickness has been determined, the shape of the cross-section can be selected which will give an edge thickness in even inches to correspond with form heights which are available or to an even half-inch if a half-inch differential is acceptable between edge thickness and form height.

Values of t_e, t_i and d which will result in balanced designs in which the edge thickening is obtained in a slope extending from the edge a distance ranging from 2 to 4 ft. can be obtained from Fig. 11. Similar information can be obtained from Fig. 12 for parabolic sections. These charts are read as indicated by the heavy dashed lines on their faces.

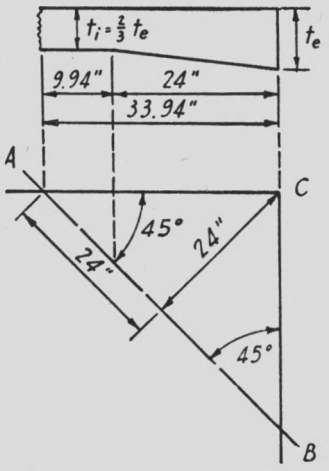


Fig. 10—Note: d = average thickness along line AC.

When thickened-edge sections are used with values of t_e and t_i determined as outlined herein, it becomes essential that provision be made for adequate load transference at all points where thin edges may occur, such as at all transverse joints and cracks and at all longitudinal joints where the slab edges are not thickened. Otherwise the thin interior portion of the pavement slab would

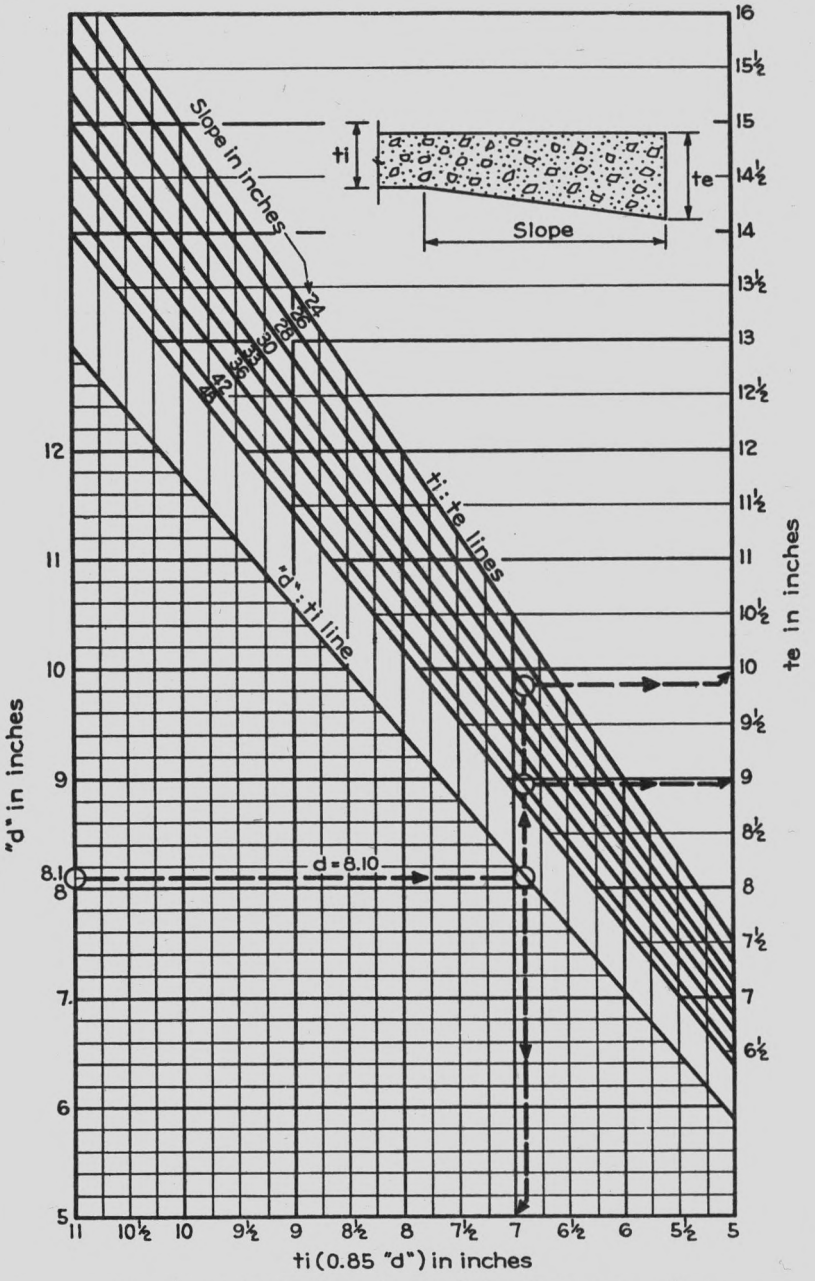


Fig. 11—Relation of *d*, ti and te for thickened-edge cross-sections.

present at such locations corners or edges of inadequate strength and the design would not be balanced. The steps which must be taken to assure load transference at edges in the thin section of the slab also will provide load transference at all corners so that Case I designs may be used. Case II has no practical application to the design of thickened-edge cross-sections.

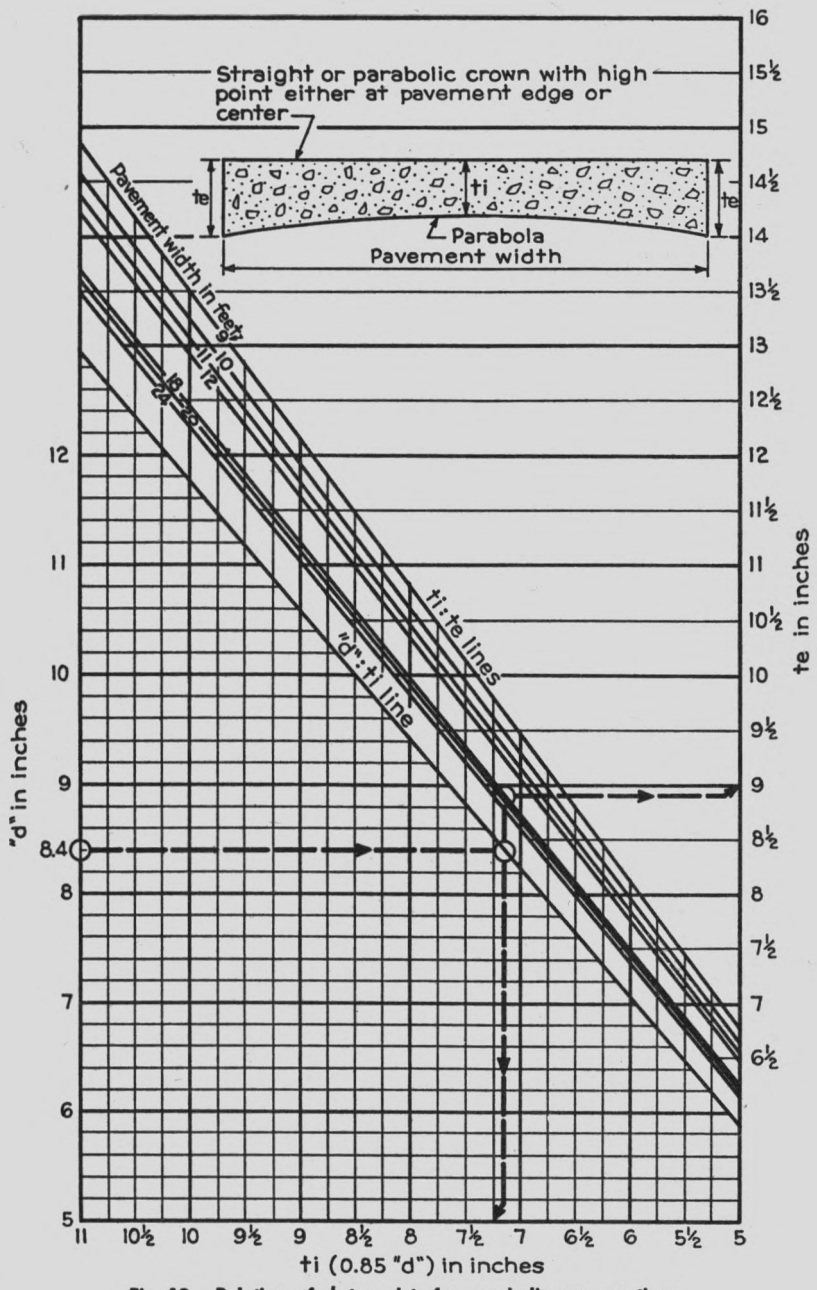


Fig. 12—Relation of *d*, t_i and t_e for parabolic cross-sections.

Table 5 presents for a variety of thickened-edge and uniform cross-sections, including those commonly used, the cubic yards of concrete required per mile as well as the effective values of *d* at exterior corners and in the slab interior. Those having the same value of *d* in both corner and interior are obviously the most economical.

Table 5—Thickened-edge and Uniform Cross-sections, Cu.Yd. Concrete Per Mile and Effective Values of *d*

Cross-section		Effective value of *d*, in.*		Cu.yd. concrete per mile		Remarks
Thickness	Slope	Exterior corners	Interior of slab	For slab 20 ft. wide	Add or subtract for each ft. that width varies from 20 ft.	
5-in. uniform		**5.00**	5.88	1,630	81.48	
6-4½-6-in.	2-ft.	**5.03**	5.29	1,516	73.33	
6-4½-6-in.	3-ft.	**5.29**	**5.29**	1,540	73.33	Balanced
7-4½-7-in.	2-ft.	5.38	**5.29**	1,548	73.33	
5½-in. uniform		**5.50**	6.47	1,793	89.63	
6-5-6-in.	4-ft.	**5.65**	5.88	1,695	81.48	
7-5-7-in.	2-ft.	**5.71**	5.88	1,695	81.48	
7-5-7-in.	2½-ft.	**5.88**	**5.88**	1,711	81.48	Balanced
7½-5-7½-in.	2-ft.	**5.88**	**5.88**	1,711	81.48	Balanced
6-in. uniform		**6.00**	7.06	1,956	97.78	
8-5½-8-in.	2-ft.	**6.38**	6.47	1,874	89.63	
7-5½-7-in.	4-ft.	**6.47**	**6.47**	1,890	89.63	Balanced
8-5½-8-in.	3-ft.	6.82	**6.47**	1,915	89.63	
6½-in. uniform		**6.50**	7.65	2,119	105.92	
8-6-8-in.	2-ft.	**6.71**	7.06	2,021	97.78	
7-in. uniform		**7.00**	8.24	2,281	114.07	
8-6½-8-in.	2-ft.	**7.03**	7.65	2,167	105.92	
8-6-8-in.	3-ft.	**7.06**	**7.06**	2,053	97.78	Balanced
9-6-9-in.	2-ft.	**7.06**	**7.06**	2,053	97.78	Balanced
9-6-9-in.	3-ft.	7.58	**7.06**	2,102	97.78	
9-6½-9-in.	2-ft.	**7.38**	7.65	2,200	105.92	
8-6½-8-in.	4-ft.	**7.47**	7.65	2,216	105.92	
7½-in. uniform		**7.50**	8.82	2,444	122.22	
9-6½-9-in.	2-ft. 8-in.	7.68	**7.65**	2,227	105.92	Approx. balanced
10-6½-10-in.	2-ft.	7.74	**7.65**	2,233	105.92	
9-7-9-in.	2-ft.	**7.71**	8.24	2,347	114.07	
8-in. uniform		**8.00**	9.41	2,607	130.37	
10-7-10-in.	2-ft.	**8.06**	8.24	2,379	114.07	
10-7-10-in.	2-ft. 4-in.	**8.24**	**8.24**	2,396	114.07	Balanced
9-7-9-in.	3-ft. 9-in.	**8.25**	**8.24**	2,404	114.07	Balanced
9-7½-9-in.	4-ft.	**8.47**	8.82	2,542	122.22	
8½-in. uniform		**8.50**	10.00	2,770	138.52	
10-8-10-in.	2-ft.	**8.71**	9.41	2,673	130.37	
10-7½-10-in.	3-ft.	**8.82**	**8.82**	2,567	122.22	Balanced
9-in. uniform		**9.00**	10.59	2,933	146.66	
10-8-10-in.	3-ft.	**9.06**	9.41	2,705	130.37	
10-8-10-in.	4-ft.	**9.29**	9.41	2,738	130.37	
11-8½-11-in.	2-ft.	**9.38**	10.00	2,852	138.52	
11-8-11-in.	2-ft. 8-in.	**9.41**	**9.41**	2,738	130.37	Balanced
9½-in. uniform		**9.50**	11.18	3,096	154.81	
11-8-11-in.	3-ft.	9.58	**9.41**	2,754	130.37	
11-9-11-in.	2-ft.	**9.71**	10.59	2,998	146.66	
11-8½-11-in.	3-ft.	**9.82**	10.00	2,893	138.52	
11-8½-11-in.	3-ft. 6-in.	**9.99**	**10.00**	2,913	138.52	Balanced
10-in. uniform		**10.00**	11.76	3,259	162.96	
12-8½-12-in.	2-ft. 6-in.	10.05	**10.00**	2,913	138.52	Approx. balanced
11-9-11-in.	4-ft.	**10.29**	10.59	3,064	146.66	
10½-in. uniform		**10.50**	12.35	3,422	171.11	
12-9-12-in.	3-ft.	**10.58**	**10.59**	3,080	146.66	Balanced
12-9½-12-in.	3-ft.	10.82	11.18	3,218	154.81	
11-in. uniform		**11.00**	12.94	3,585	179.26	
12-9½-12-in.	4-ft.	**11.12**	11.18	3,259	154.81	Approx. balanced
12-10-12-in.	4-ft.	**11.29**	11.76	3,390	162.96	
11½-in. uniform		**11.50**	13.53	3,748	187.40	
13-10-13-in.	3-ft.	**11.58**	11.76	3,406	162.96	
14-10-14-in.	2-ft. 6-in.	**11.77**	**11.76**	3,422	162.96	Balanced
13-10-13-in.	3-ft. 6-in.	11.79	**11.76**	3,430	162.96	Approx. balanced
12-in. uniform		**12.00**	14.12	3,911	195.55	

*Smaller value of *d* for each section is shown in bold face type.

Table 6 gives the average thickness of the cross-sections and the value of *d* for both edges and interior of a number of cross-sections with parabolic subgrade. The sections shown are balanced or nearly balanced but the parabolic cross-section requires more concrete than the balanced sections of the same load capacity shown in Table 5. For example, the 8-6½-8 parabolic section 22 ft. wide is balanced with both center and edge having an effective *d* of 7.65 in. This section would require 2510 cu.yd. of concrete per mile of pavement. The 9-6½-9, 22-ft. cross-section with 2-ft. 8-in. slope in Table 5 is approximately balanced with effective values of *d* at both edge and interior at least equal to that of the 8-6½-8, 22-ft. parabolic section. This cross-section requires 2439 cu.yd. per mile of pavement, 71 cu.yd. less than the parabolic cross-section.

A uniform-thickness pavement would have to be 7.65 in. thick to have edge strength equivalent to the preceding cross-sections. It would require 2743 cu.yd. per mile of pavement—from 233 to over 300 cu.yd. more than the above thickened-edge sections of equivalent load-carrying capacity.

A difference of opinion exists among engineers as to the relative merits of concrete pavement slabs of uniform thickness compared with those having thickened edges. Many believe that the greater economy of the thickened-edge design justifies the additional care and labor required in preparing the subgrade. However, the uniform-thickness slab is favored by other engineers who like its greater simplicity of construction. Either type of cross-section will give good service if properly designed and constructed.

Unbalanced sections are not as efficient in use of materials as the balanced sections. They should be used only when other considerations offset their lack of economy. For unbalanced sections, the smaller value of *d* must be used in the stress analysis or in selection of a section having the *d* required for the loading and subgrade conditions.

Fatigue of Concrete

Concrete, like other structural materials, is affected more by repeated critical stresses than by a single stress of the same magnitude. This effect, for want of a better name, has been called "fatigue".

There has been considerable research on fatigue behavior of concrete. Some has dealt with repetitions of compressive stress; other studies have investigated the behavior of concrete under repetitions of flexural or bending stress.

All researchers are in practical agreement on the following facts:

1. When the repeated stress does not exceed 50 per cent of the ultimate strength (safety factor* not less than 2) the concrete will stand an unlimited number of stress repetitions without failure.

2. When the repeated stress is lower than 50 per cent of the ultimate strength

*"Factor of safety" means the value obtained by dividing the modulus of rupture, or ultimate breaking strength of the concrete in flexure, by the stress produced by a given load.

$$\text{Safety factor} = \frac{\text{modulus of rupture in psi}}{\text{unit stress in psi}}.$$

Table 6—Typical Parabolic Cross-sections, Effective Values of *d* and Average Thicknesses

Cross-section, in.	Effective value *d* in interior, in.	Effective value of *d* at exterior corners, in in. for various pavement widths									Average thickness, in.*
		9 ft.	10 ft.	11 ft.	12 ft.	12½ ft.	18 ft.	20 ft.	22 ft.	24 ft.	25 ft.	
6-4½-6	5.29	5.25	5.31	5.36	5.40	5.42	5.58	5.62	5.65	5.67	5.69	5.00
6-5-6	5.88	5.50	5.54	5.57	5.60	5.62	5.72	5.74	5.76	5.78	5.79	5.33
7-5-7	5.88	6.01	6.08	6.15	6.21	6.23	6.44	6.49	6.53	6.57	6.58	5.67
7-5½-7	6.47	6.25	6.31	6.36	6.40	6.42	6.58	6.62	6.65	6.67	6.69	6.00
7-6-7	7.06	6.50	6.54	6.57	6.60	6.62	6.72	6.74	6.76	6.78	6.79	6.33
8-5½-8	6.47	6.76	6.85	6.93	7.01	7.04	7.30	7.36	7.41	7.46	7.48	6.33
8-6-8	7.06	7.01	7.08	7.15	7.21	7.23	7.44	7.49	7.53	7.57	7.58	6.67
8-6½-8	7.65	7.25	7.31	7.36	7.40	7.42	7.58	7.62	7.65	7.67	7.69	7.00
9-6½-9	7.65	7.76	7.85	7.93	8.01	8.04	8.30	8.36	8.41	8.46	8.48	7.33
9-7-9	8.24	8.01	8.08	8.15	8.21	8.23	8.44	8.49	8.53	8.57	8.58	7.67
9-7½-9	8.82	8.25	8.31	8.36	8.40	8.42	8.58	8.62	8.65	8.67	8.69	8.00
10-7½-10	8.82	8.76	8.85	8.93	9.01	9.04	9.30	9.36	9.41	9.46	9.48	8.33
10-8-10	9.41	9.01	9.08	9.15	9.21	9.23	9.44	9.49	9.53	9.57	9.58	8.67
10-8½-10	10.00	9.75	9.31	9.36	9.40	9.42	9.58	9.62	9.65	9.67	9.69	9.00
11-8-11	9.41	9.51	9.62	9.72	9.81	9.85	10.16	10.23	10.29	10.35	10.37	9.00
11-8½-11	10.00	9.76	9.85	9.93	10.01	10.04	10.30	10.36	10.41	10.46	10.48	9.33
11-9-11	10.59	10.01	10.08	10.15	10.21	10.23	10.44	10.49	10.53	10.57	10.58	9.67
12-9-12	10.59	10.51	10.62	10.72	10.81	10.85	11.16	11.23	11.29	11.35	11.37	10.00
12-9½-12	11.18	10.76	10.85	10.93	11.01	11.04	11.30	11.36	11.41	11.46	11.48	10.33
12-10-12	11.76	11.01	11.08	11.15	11.21	11.23	11.44	11.49	11.53	11.57	11.58	10.67

* Cu.yd. concrete per mile = 16.296 × average thickness in in. × width in ft.

(safety factor greater than 2) the repetition of stress is actually beneficial and strengthens the concrete.

3. When the repeated stress materially exceeds 50 to 55 per cent of the ultimate strength (safety factor less than 2) continued repetition of stress will cause failure or breaking of the concrete.
4. When the safety factor ranges between 1 and 2 the number of repetitions required to cause failure varies, the number decreasing as the safety factor decreases.
5. When there is a period of recovery between stress applications the fatigue action is diminished.
6. There is a difference in the fatigue behavior of concrete under repetitions of compressive and flexural stress, the action under flexural stress being the less severe for the same percentage of the ultimate strength.

In concrete subjected to flexural stress, the compressive and tensile stresses in the outer fibers of the member are approximately equal. The tensile strength of concrete is much less than its compressive strength. When the load or bending moment increases so that rupture of the concrete is approached, the resulting tensile stress reaches a high percentage of the ultimate tensile strength of the concrete while the compressive stress on the other side of the beam or slab is still a relatively small percentage of the ultimate compressive strength of the concrete.

Therefore, when flexural stresses greater than 50 per cent of the ultimate modulus of rupture of the concrete are repeated, the concrete becomes fatigued in tension while the compressive stress remains well below the point at which fatigue in compression would result. From this it follows that the fatigue under repetitions of flexural stress is the governing factor.

Fig. 13 shows the fatigue behavior of concrete in flexure. The relationship between the safety factor and the number of stress repetitions required to induce failure of the concrete, as given here, represents the best data available from published reports of research. The curve is in agreement with Illinois Highway Department studies, which are the most extensive studies available on the fatigue behavior of full-sized concrete specimens under repetitions of flexural stress.

For design purposes this curve is safe because it is based on repetitions of stress at relatively high frequency, 10 to 40 per minute, while on a road or street there will be, almost invariably, a substantial period of time for recovery of the concrete between applications of the successive heavy loads. Experience has proven that this period of recovery between loadings reduces the fatigue effect.

Application of Fatigue Principle—The method of designing concrete pavement slabs presented in this manual applies, practically and usefully the knowledge of the fatigue behavior of concrete discussed in the preceding section. Pavement designed by this method is capable of carrying an unlimited number of wheel loads equal to or less than that for which it was designed as well as the occasional heavier wheel loads that are sure to use it.

Fig. 13—Fatigue of concrete in flexure.

To give satisfactory service, all pavement slabs should be designed to carry the controlling wheel load* with a safety factor of at least 2. For the less frequent loads heavier than the controlling wheel load, the safety factor to be used with each should be of such value that the pavement will have adequate structural life when subjected to the anticipated volume of these heavier loads.

The useful life of a concrete pavement may safely be estimated at 25 to 30 years**, or in the case of a pavement intended for temporary use, a shorter period may be used. Using conservative traffic predictions, the anticipated number of wheel loads of each weight group that will use the pavement during its useful life can be computed.

When designing a two-lane pavement it is reasonable to assume, under normal traffic conditions, that the traffic in each direction will be equal. Therefore, when determining the number of wheel load repetitions for each weight group per traffic lane, only one-half the total*** reported or estimated traffic for each group should be used. Where an unbalanced traffic condition exists, the half of the pavement carrying the heaviest traffic (weight and volume of trucks) may require a heavier design than the half carrying the lighter traffic. Unless the traffic is highly unbalanced, both lanes of a two-lane pavement are usually designed to carry the same traffic as the most heavily traveled lane.

When designing a multiple-lane (4 or more lanes) pavement, it must be remembered that the heavier trucks almost invariably travel in the outer lanes. They normally use the inner lanes only while passing slower vehicles. Because of this it is common practice among designers to assume that all commercial traffic is concentrated in the outer lanes, and that the inner lanes carry only the lighter, faster moving traffic. Therefore, when determining the number of wheel load repetitions for each weight group per outer traffic lane, use one-half of the total reported or estimated traffic for the group. Usually all lanes of a multiple-lane pavement are designed to carry the same traffic as the more heavily traveled outer lanes. However, some saving may be effected by taking this unbalanced traffic into account and designing each lane for the traffic it will have to carry. Thus, the outer lanes will have a heavier design than the inner lanes and an economical design will be achieved.

The actual design procedure that applies the knowledge of the fatigue behavior of concrete under repeated stresses may be outlined as follows:

1. Determine the anticipated number of wheel loads (by weight groups) that will use the traffic lane during the life of the pavement.
2. Compute the controlling wheel load using these data.
3. Obtain the working stress of the concrete that is to be used with this wheel load by dividing the modulus of rupture of the concrete by a safety factor of 2.

*"Controlling wheel load" is taken as the average of the heaviest 100,000 anticipated wheel loads that will use a traffic lane during the life for which the pavement is being designed.

**Farrell and Paterick, "Life Characteristics of Highway Surfaces", *Proceedings*, Highway Research Board, 1948, Vol. 28, page 40.

***Traffic volumes (auto, commercial or by weight groups—see Table I) as reported by Statewide Planning Surveys, etc., for total traffic (sum of traffic in both directions). Seldom is the traffic in each direction reported separately.

4. Determine the thickness of pavement that will be required to carry an unlimited number of wheel loads equal to or less than the controlling wheel load by using the appropriate formula or design chart.
5. Use the pavement thickness thus obtained as an indicated thickness to select a cross-section (proportioned by methods previously described) that should be adequate.
6. Then check the selected cross-section to make certain that it will be adequate to carry the anticipated traffic. Ordinarily pavements designed by this method will prove capable of carrying not only an unlimited number of wheel loads equal to or less than the controlling wheel load, but also the less frequent heavier wheel loads that will use them.

The foregoing method of designing concrete pavements is illustrated by the step-by-step example which follows.

PROCEDURE
For Pavement Slab Design

Before starting the actual design problem, basic data pertaining to the factors influencing the design must be determined, evaluated and tabulated. These essential preliminary steps are explained in the following paragraphs:

Preliminary Steps

1. Prepare a table showing the anticipated traffic (volumes and wheel load distribution). This may be done by one of the following methods:
   a. Make a special traffic survey for the project. This to include traffic counts, loadometer studies, origin and destination studies and a review of the general environment, adjacent industries, etc., as a basis for projecting estimates of future traffic. A discussion of traffic surveys is given on page 5.
   b. If only a general traffic appraisal can be made, use data developed by statewide and local traffic surveys on comparable roads or streets as the basis for an estimate of future traffic.
   c. If data from statewide and local traffic surveys are not available, data for the appropriate class of road may be taken from Table 1. This is recommended only as a last resort for, wherever possible, designs should always be based upon traffic expectancy for the actual location under consideration. It is suggested that the format used in Table 1 be followed when assembling the traffic data.
2. Determine the value of the subgrade modulus *k*. This may be determined by one of the methods given on page 8.
3. Decide upon the life for which the pavement is to be designed. Except for temporary pavements which are to be used for only short periods, a life of 30 years is usually used.

4. Establish the modulus of rupture of the concrete that is to be used. This is commonly taken at 700 psi, though concrete with more or less strength may be found more economical (see page 13).

Example of Design Problem

Design of Cross-Section for Class II Highway, Primary Route in a Rural Area

The actual design can best be demonstrated by following through an example, with an explanation of each step. The procedure is illustrated and calculations facilitated by tabulating all data, see Table 7.

1. Prepare in blank form, a table similar to Table 7.
2. At the top of the table enter the particular "case", "design life" in years, "subgrade modulus k", and "modulus of rupture of concrete" in their proper places. In this example Case I is used; the values of the other factors used are shown at the top of the table.
3. Enter in column 1 the wheel load groups as shown. The maximum weight for each load group is increased by a 20 per cent impact factor and the value entered in column 2. These gross wheel loads are used in later phases of the problem.
4. In column 3 enter the anticipated number of wheel load repetitions per day per traffic lane for each wheel load group. For two-lane pavements or the outer lanes of multilane pavements, the values entered in column 3 are usually obtained by taking the number of wheel loads shown for each weight group in the previously prepared traffic table (similar to Table 1) and dividing by two. The reason for this is explained on page 33. For two-lane pavements with unbalanced traffic or for the inner lanes of multilane pavements, the anticipated traffic on each lane is estimated and a separate design worked out for each lane. The traffic data used in this example problem were taken from Table 1. The values in column 3 are the average values for a Class II highway divided by two.
5. Multiply the values in column 3 by the number of days in the design life of the pavement (in the example, 365 x 30 = 10,950), and record the results in column 4. The values thus obtained represent the total number of load repetitions for each wheel load group that are anticipated during the design life of the pavement. To eliminate unnecessary work, these computations are started at the bottom of the column and carried upward. As soon as it is evident that the remaining lighter wheel load groups will produce values in excess of 100,000, there is no need of continuing the calculation.
6. Compute the controlling wheel load (see footnote *, page 33), recording the calculations in column 5. The computation is made in the following manner:
   a. Determine the number of load repetitions per day that will produce 100,000 repetitions during the design life of the pavement. This may

Table 7—Pavement Design for Class II Highway, Primary Route in a Rural Area

Case I—Protected corners Design life = 30 years

Subgrade modulus, k = 100 psi per in.; Modulus of rupture of concrete = 700 psi.

1	2	3	4	5	6	7	8	9
Wheel load lb.		Load repetitions			Analysis of design selected 10-7-10 with 2-ft. 4-in. slope d = 8.24 in.			
By groups, static	Maximum for group plus 20 per cent impact	Number per day, one direction	Anticipated for 30 year period	Determination of "Controlling Wheel Load" and indicated d	Actual stress, psi.	Actual safety factor	Actual allowable number of stress repetitions	Fatigue resistance consumed by each load group, per cent
under 4,000	4,800	703	over 100,000		under 350	over 2.0	unlimited	
4,000— 5,000	6,000	54	over 100,000		under 350	over 2.0	unlimited	
5,000— 6,000	7,200	51	over 100,000		under 350	over 2.0	unlimited	
6,000— 7,000	8,400	60	over 100,000		under 350	over 2.0	unlimited	
7,000— 8,000	9,600	64	over 100,000		under 350	over 2.0	unlimited	
8,000— 9,000	10,800	61	over 100,000		under 350	over 2.0	unlimited	
9,000—10,000	12,000	30	over 100,000		under 350	over 2.0	unlimited	
10,000—11,000	13,200	10	109,500	11,000 × 4.4 = 48,400	under 350	over 2.0	unlimited	
11,000—12,000	14,400	3.4	37,230	12,000 × 3.4 = 40,800	347	2.02	unlimited	
12,000—13,000	15,600	1.0	10,950	13,000 × 1.0 = 13,000	373	1.87	56,000	20
13,000—15,000	18,000	0.3	3,285	15,000 × 0.3 = 4,500	421	1.66	22,000	15
				9.1 106,700				35 Total (for wheel loads having S.F. less than 2.0)

$\frac{106,700}{9.1}$ = 11,725 lb. = controlling wheel load

11,725 × 1.2 = 14,070 lb. = gross controlling wheel load (includes 20% impact).

Indicated d = 8.15 in.

be computed or read direct from Fig. 14. A value of 9.1 was obtained for the 30-year design life used in this example $\left(\frac{100{,}000}{30\times365}=9.1\right)$.

b. Compute the weighted average of the heaviest wheel loads (using maximum for each group in column 1) that will produce this number (9.1) of repetitions per day. The weighted average thus obtained is the controlling wheel load. It is computed as indicated in column 5. The wheel load for the heaviest group is multiplied by the corresponding value from column 3 ($15{,}000\times0.3=4500$), recording the computation as shown. This operation is repeated for each succeeding lighter group until the sum of the values taken from column 3 is just under the value (9.1) obtained in step 6*a* above. The maximum wheel load for the next lighter group is multiplied by a number just sufficient to make the total of these multipliers equal the value (9.1) obtained in step 6*a*. The products of these multiplications are totaled and divided by the sum of the multipliers (9.1). The answer thus obtained is the weight, in pounds, of the controlling wheel load.

7. Determine the indicated *d* (the thickness of pavement that will be required to carry an unlimited number of wheel loads equal to or less than the controlling wheel load) and record the value in column 5. This is done by using the design chart (Figs. 4, 5, 6 or 7) that corresponds to case and wheel load being used. In order to use these charts, which are based on Formulas 1 and 2, the gross controlling wheel load must be determined and used (see page 17). The gross controlling wheel load is the static controlling wheel load (obtained in step 6*b* above) increased by an impact factor of 20 per cent, in this example $11{,}725\times1.2=14{,}070$ lb. The indicated *d* is read from the proper design chart (Fig. 4, Case I, dual tires in this example) by using the working stress of the concrete (modulus of rupture divided by safety factor of 2, $\frac{700}{2}=350$ psi), the subgrade modulus *k* (100 psi per in.) and the gross controlling wheel load (14,070 lb.). The design chart (Fig. 4) is read by entering it at the left side at the working stress (350) and moving horizontally until the proper *k* value (100) is intersected; then move vertically upward until the gross controlling wheel load (14,070) is intersected; thence horizontally to the right where the value of the indicated *d* in inches (8.15) is read. This procedure is illustrated on the face of Fig. 4 by a heavy dashed line; for a working stress of 325, a *k* of 100 and a gross controlling wheel load of 11,000, an indicated *d* of 7.48 is read.

8. Having determined the indicated *d*, select a practical pavement cross-section that will be adequate and economical for the specific conditions. Cross-sections having a uniform thickness or thickened-edge section may be selected directly from Table 5 and cross-sections having a parabolic section may be selected directly from Table 6. When selecting a cross-section from Tables 5 or 6, make certain that the lowest effective

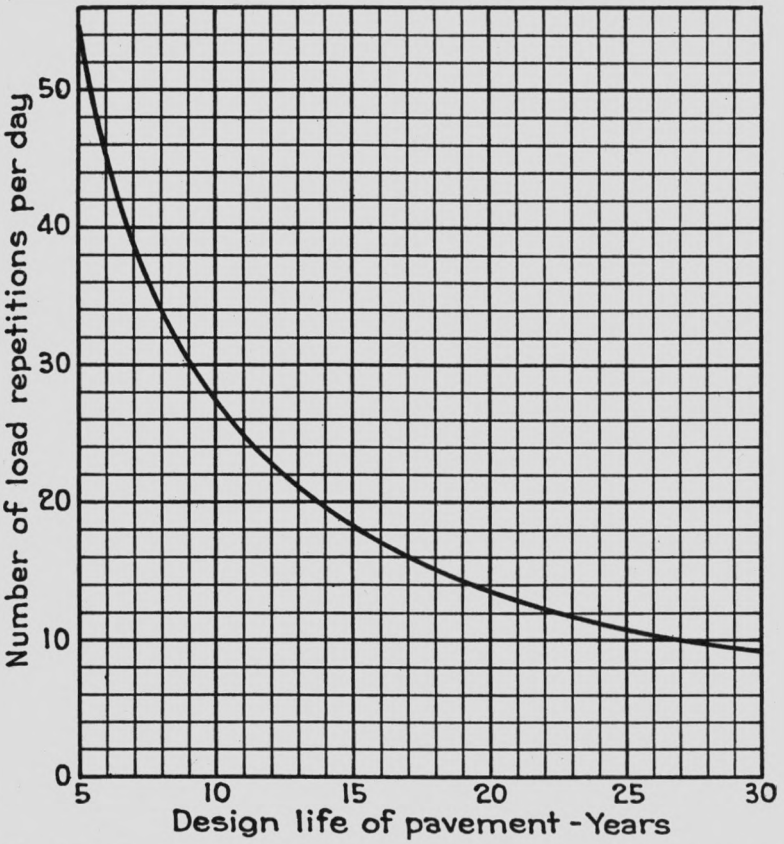


Fig. 14—Load repetitions per day that will produce 100,000 stress repetitions during design life of pavement.

value of *d* listed for that section is not less than the indicated *d* obtained in step 7. Cross-sections having thickened-edge sections may also be determined by use of Fig. 11 and those having parabolic sections, by use of Fig. 12. In the example problem a 10-7-10 thickened-edge cross-section having a 2-ft. 4-in. slope was selected from Table 5. This section was chosen because it made more economical use of concrete than would either a 9-7-9 parabolic or an 8¼-in. uniform-thickness section (see Table 8, Class II highway).

9. Analyze the cross-section selected to make certain that it will carry not only the gross controlling wheel load and all lighter wheel loads with a safety factor of 2.0 or more, but that it will also carry the less frequent heavier wheel loads without unduly fatiguing the concrete. This analysis is made by the method illustrated in columns 6 through 9 of Table 7 and is explained on the page following.

a. In column 6 record the actual stress (flexural) that each gross wheel load (column 2) will produce in a slab having the cross-section selected. This stress may be computed by formulas 1 and 2 or read from the proper design chart (Figs. 4, 5, 6 or 7). The method of determining the actual stress when the *d*, gross wheel load and *k* are known is indicated by the heavy dashed line on the face of the chart. To illustrate, using the values for this problem: enter the chart (Fig. 4) at the right margin at the value of the effective *d* (8.24) of the selected section, move horizontally until the heaviest gross wheel load from column 2 (18,000) is intersected, thence vertically downward until the proper *k* value (100) is intersected, thence horizontally to the left margin where the flexural stress (421) is read. This operation is repeated for each succeeding lighter gross wheel load listed in column 2 (*d* and *k* will remain constant for any one analysis) until the actual stress determined is less than the working stress for unlimited operations (350 psi). These stress determinations need not be continued, as all lighter gross wheel loads will produce actual stresses less than the working stress (350), and thus will produce no fatigue in the concrete as their safety factors will be larger than 2.0.

b. In column 7 record the safety factor that each of the stresses listed in column 6 will have. The safety factors are obtained by dividing the modulus of rupture of the concrete (700) by each stress value in column 6 $\left(\frac{700}{421}=1.66\right)$. When the actual stress is below the working stress (under 350) the safety factor is over 2.0 and no further computation need be made.

c. In column 8 record the actual allowable number of stress repetitions that each safety factor, listed in column 7, will permit without unduly fatiguing the concrete. These values are read directly from Fig. 13 (1.66 giving 22,000).

d. In column 9 record the percentage of the total fatigue resistance of the selected section that will be consumed by each wheel load group having a safety factor less than 2.0. This percentage is obtained by dividing the values from column 4 (total load repetitions anticipated during the design life of the pavement) by the values in column 8 (actual allowable number of stress repetitions) and expressing as a per cent $\left(\frac{3{,}285}{22{,}000}\times 100=15 \text{ per cent}\right)$. The individual percentages thus computed are added together to obtain the total percentage (35 per cent) of the fatigue resistance of the selected design that will be used during the design life of the pavement by the anticipated traffic.

The thickened-edge design selected (10-7-10 with 2-ft. 4-in. slope) in this example is well within the limits of safe design since only 35 per cent of its fatigue resistance is used up by the anticipated traffic.

Theoretically the limit of fatigue resistance consumed by the anticipated

traffic could be 100 per cent. On all except the most heavily traveled roads it is safe to exceed this limit by approximately 20 per cent. The reasons for this are that not all the wheel loads in a group will be equal to the heaviest for that group as was assumed; the concrete throughout most of its life will have a strength well above the 28-day strength used in the design; and the fact that the load repetitions are distributed over a period of years with a considerable time for recovery between loads (especially between the critical heavier loads) reduces the effect of fatigue. When designing heavily traveled highways, such as Class I or II which generally carry a large number of the heavier wheel loads, the fatigue resistance consumed by the anticipated traffic should not exceed 100 per cent.

In event that the design selected proves inadequate when analyzed as described, a section with a slightly higher effective *d* should be selected. This new design should then be analyzed in the same manner as the original design. When this is done it is only necessary to repeat the analysis (columns 6 through 9 of Table 7).

Table 8 was prepared using the design method just described. It shows the pavement cross-sections that would be adequate for the traffic volumes and wheel load distributions (average values) for each of the four classes of highways given in Table 1, when all other conditions are equal to those used in the preceding example problem. Table 8 shows the effect of major traffic variations on the thickness of pavement required. It also permits a comparison of the three principal types of cross-sections as to their effective *d* and the cubic yards of concrete each would require per mile for each of the four classes of highways. It will be noted that when selecting the uniform-thickness sections shown in this table, the thickness was taken at the next even half-inch above the indicated value of *d*. This procedure should always be followed when designing uniform-thickness sections for the reasons previously given on page 25.

Table 8—Summary and Comparison of Designs That Would Be Adequate for Average Traffic Distributions Given in Table 1*

Class of highway	Indicated *d* in.	Pavement cross-section		Width ft.	Effective *d* in.	Concrete per mile cu.yd.
		Thickness in.	Type of section			
I		9	Uniform thickness	24	9.00	3,520
Primary route in or near a metropolitan area	8.64	10-7½-10	Thickened edge with 3-ft. slope	24	8.82	3,056
		9-7½-9	Parabolic	24	8.67	3,129
II		8½	Uniform thickness	24	8.50	3,324
Primary route in a rural area	8.15	10-7-10	Thickened edge with 2-ft. 4-in. slope	24	8.24	2,852
		9-7-9	Parabolic	24	8.24	3,000
III		7½	Uniform thickness	22	7.50	2,688
Lightly traveled primary route	7.47	9-6½-9	Thickened edge with 2-ft. 8-in. slope	22	7.65	2,439
		8-6½-8	Parabolic	22	7.65	2,509
IV		7½	Uniform thickness	20	7.50	2,444
Secondary or county trunk route	7.35	9-6½-9	Thickened edge with 2-ft. slope	20	7.38	2,200
		8-6½-8	Parabolic	20	7.62	2,281

*Note: All designs based on Case I—protected corners; subgrade modulus, k = 100 psi per in. and a pavement design life of 30 years.

SUBBASES AND SUBGRADE TREATMENTS
Under Concrete Pavements

Subbases

Research on experimental pavement slabs and on concrete pavements in service has established the fact that a loaded concrete pavement imposes very low unit pressures on the subgrade. For this reason, it is unnecessary and uneconomical to build up the supporting capacity of the subgrade with thick ballast courses. These thick granular subbases, which are an essential part of nonrigid pavements, have no justification under concrete pavements as they add to the cost without making a corresponding addition to its capacity and serviceability.

This does not mean that subbases are not, under some conditions, needed under concrete pavements. But they should be used only where they will correct or counteract an unsatisfactory soil or unstable soil condition. They are needed if pavement damage from one or more of the following causes is anticipated or known to exist:

1. Frost action.
2. Swell and shrinkage in high-volume-change soils.
3. Pumping of fine-grained soils.

Their use for the sole purpose of increasing subgrade support (k value) in order to reduce the slab thickness is rarely, if ever, justified. It is generally more economical to design the slab for the existing subgrade-bearing capacity than to raise the subgrade support by using a thick subbase. Selective grading, placing the better soils from the cuts in the upper foot or two of the grade, should be done wherever practical and economical. In such cases, advantage should be taken of the increase in k value that will result.

Prevention of Damage from Frost Action—Frost action may cause severe local (differential) heaving which will destroy the utility of the pavement surface, or it may cause a progressive expansion and softening of the subgrade until its supporting value is reduced to a point that the pavement can no longer carry its normal loads.

Materials susceptible to frost action and the conditions under which detrimental frost action take place are covered in detail in many of the textbooks on soils and soil mechanics*. Soils of high capillarity, such as silts and very fine sands, are most susceptible to frost action. A constant source of free water, such as high groundwater, must be within the range of capillary activity before detrimental frost action can occur in these soils. If free water is not available, there will be no frost heave. Soils and conditions conducive to frost action are easily recognized by trained soil personnel.

Differential heaving of pavements occurs where frost-susceptible soils are

*C. A. Hogentogler, *Engineering Properties of Soil*, New York, McGraw-Hill Book Co., pages 146 to 154.

pocketed in less affected soils. Where this condition exists, excavation, drainage and backfill with non-frost-heaving granular materials are the solution.

Where groundwater is encountered in such a preventive operation, the backfilling should be done with open-graded (free-draining) material and provision made for drainage. If groundwater is not a problem, the more densely graded non-frost-heave materials may be used as backfill. Such soils do not readily admit surface water and extensive provisions for drainage are not required.

Granular materials containing less than 3 per cent by weight of particles smaller than 0.02 mm., that meet the grading and plasticity requirements given for "open-graded" materials in Figs. 15, 16, 17 or 18 have given satisfactory service when used as backfill or subbase to prevent frost damage.

To prevent damage caused by differential heaving, subgrade removal and replacement are usually carried to a depth equal to about ½ of frost penetration. If the natural subgrade soil differs considerably from the non-frost-heaving material used in backfilling, the replacement should be feathered out so as to prevent an abrupt change in subgrade support. Where uniform heaving is anticipated or in areas of deep frost penetration, such as the northernmost states, the removal and replacement, or blanketing, with open-textured granular material need only be ⅛ to ½ the depth of frost penetration. Whenever free-draining materials are placed in or on the subgrade to prevent damaging frost action, some positive means of drainage must be provided.

Control of Swell and Shrinkage in High-Volume-Change Soil*—Concrete pavements in *semi-arid regions*, when placed on heavy clay soils that are in a dry condition, are likely to develop rough riding surfaces during rainy seasons. Their uneven surfaces are characterized by high joints and loss or gain in crown. This warping of the pavement is caused by differential swell or shrinkage of the subgrade soil. High joints and loss of crown occur only where heavy clay soils are compacted when relatively dry, or where they are allowed to dry before placing of the slab. The dry soil then swells as a result of absorption of water which enters the subgrade through leaky transverse joints or cracks and along the edge of the pavement. Gain in crown occurs only where similar soils are covered with a slab when they are in a very wet condition and the soil then shrinks due to drying out at the slab edges. The slab edges are thus left unsupported.

Soil types which are conducive to excessive swell and shrinkage and the conditions under which these occur may be identified by means of established test methods** or by experience with similar soils under similar conditions. Soils having a swell of 2 per cent or less need no special treatment even in dry climates. In dry climates, soils which show a swell of 2 to 4 per cent require only control of moisture content and compaction***. The highly expansive soils (over 4 per

*See footnote **, page 14.

**Allen and Johnson, "The Results of Tests to Determine the Expansive Properties of Soils", Highway Research Board *Proceedings*, 1936, Vol. 16, page 220; or AASHO Designation T116-49.

***The swell of a soil compacted to the usual requirement of 90 to 95 per cent of standard density (AASHO Designation T99-49) is at a minimum if the soil is compacted at a moisture content slightly in excess of optimum and it is not permitted to dry out prior to the placement of the pavement.

cent swell) in dry climates require the use of a blanketing course in addition to control of moisture content and compaction. Even these highly expansive soils do not produce rough riding pavements in climates free from prolonged dry periods; therefore, steps to control warping are necessary only in arid and semi-arid regions.

Thickness of nonexpansive (usually granular) subbases required to prevent warping depends upon the expansive properties of the underlying soil, its moisture content and density, the climate of the locality, and upon the watertightness of the subbase material selected.

Densely graded subbases 4 in. in thickness and open graded subbases 4 to 12 in. in thickness have prevented warping when they were placed on subgrades constructed under adequate moisture and density control and the grade had not dried out prior to their placement.

Available data indicate that satisfactory results are not generally obtained by intermixing granular material with expansive soils. This can be attributed to two factors: (1) difficulty encountered in pulverizing the heavy clay soils at their normal field moisture contents; and (2) incomplete mixing of the granular materials with the clay soils for the full intended depth of treatment.

Prevention of Pumping of Fine-Grained Soils—When certain types of fine-grained subgrade soils are highly saturated, the repeated deflections of pavement slab ends cause a *mixture of soil and water* to be ejected at or near joints and cracks. This action is known as "pumping". Three factors must be present to produce pumping:

1. A soil that will go into suspension.
2. Free water.
3. Frequent passage of heavy wheel loads.

If one of these is absent, pumping will not occur. Continued pumping soon leads to the removal of sufficient subgrade soil so that the slab ends are left unsupported. In the design of concrete pavements that are to be placed on fine-grained soils, the prevention of pumping should be given careful consideration.

Extensive research* has shown that: (a) the subgrade soils conducive to pumping are those in which the silt and clay fractions predominate; (b) pumping is most severe on soils having high clay contents; and (c) that pavements placed on subgrades of nonplastic granular soils, such as sands and gravels, do not pump.

These studies have also shown that:

1. No pumping occurred on fairly well graded natural soils, granular subbases and mulches containing more than 55 per cent sand and gravel (material retained on the No. 200 sieve) when the pavement carried a moderate volume of heavy wheel loads.
2. Even under the heaviest loadings no pumping occurred on fairly well

*"Report of Committee on Maintenance of Joints in Concrete Pavements as Related to the Pumping Action of Slabs", Highway Research Board *Proceedings*, 1948, Vol. 28, page 281.

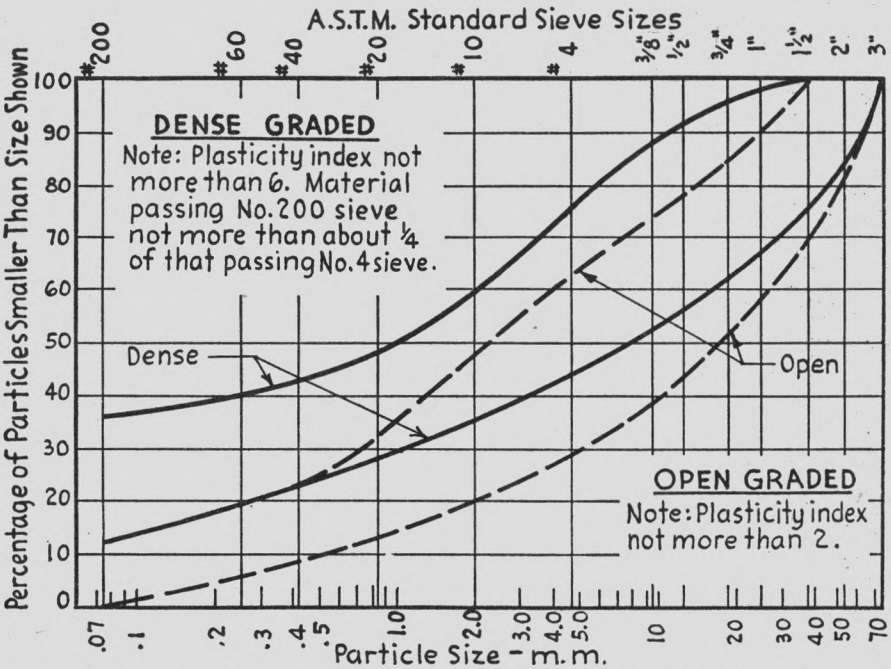


Fig. 15—Gradation limits of granular materials (1½-in. to 3-in. max. size) that have given good service as subbases under concrete pavements.

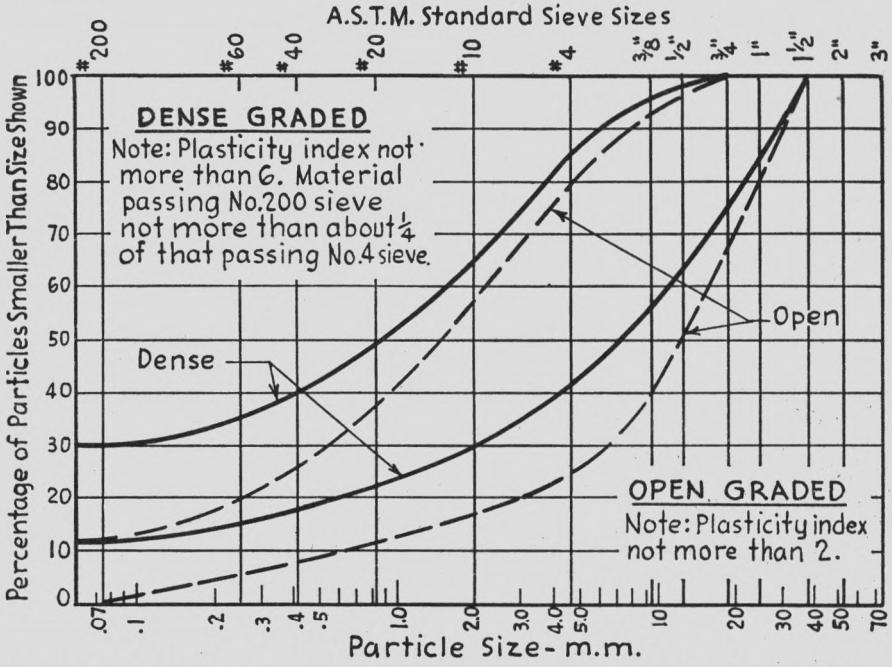


Fig. 16—Gradation limits of granular materials (¾-in. to 1½-in. max. size) that have given good service as subbases under concrete pavements.

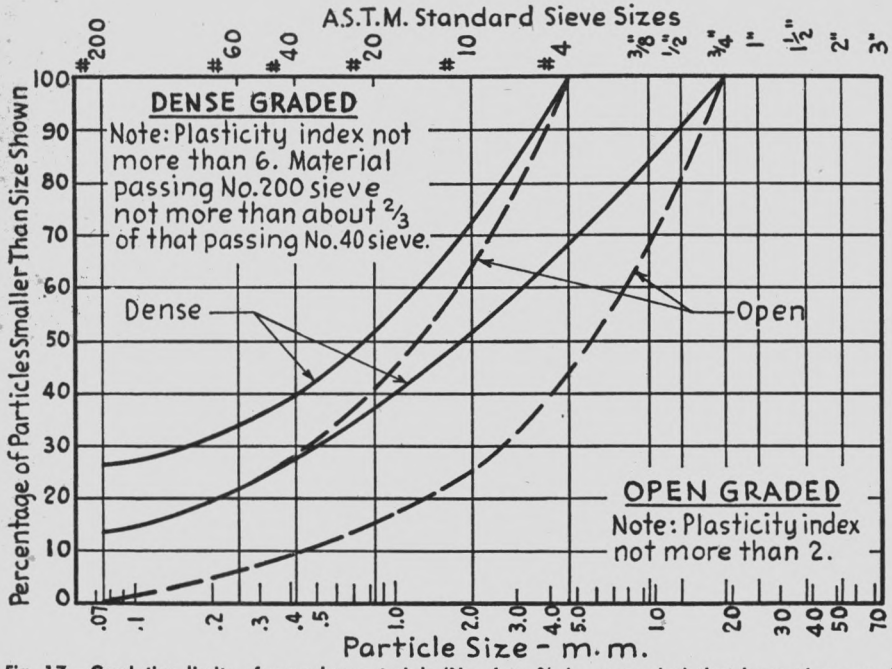


Fig. 17—Gradation limits of granular materials (No. 4 to ¾-in. max. size) that have given good service as subbases under concrete pavements.

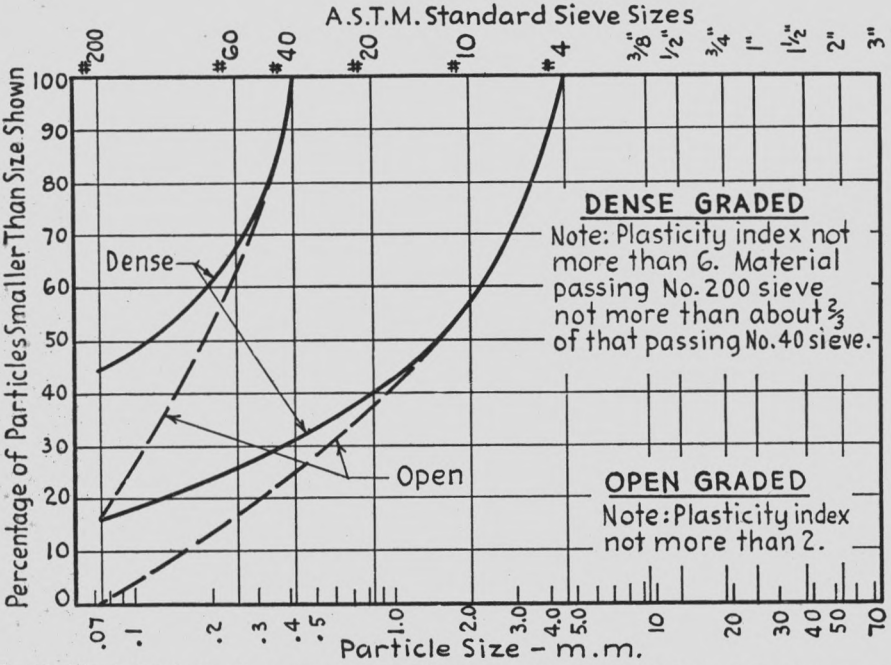


Fig. 18—Gradation limits of granular materials (No. 40 to No. 4 max. size) that have given good service as subbases under concrete pavements.

graded granular soils (more than 55 per cent retained on a No. 200 sieve) whose minus No. 40 sieve fraction had a plasticity index (P.I.) of 6 or less.

3. Granular subbases, 4 to 6 in. in thickness, that meet the grading and plasticity requirements shown in Figs. 15, 16, 17 or 18 have prevented pumping even under adverse conditions.
4. Water which had infiltrated through the joints and cracks and along the pavement edges was invariably the source of the free water which caused pumping. Proper construction and maintenance practices reduced but did not completely prevent the infiltration of water.
5. Various features of pavement design aid in reducing or preventing pumping. Foremost of these is the jointing of the pavement. The omission of expansion joints when combined with a close spacing (15 to 20 ft.) of contraction joints has been found to materially reduce the tendency of a slab to pump.

Subbase Materials

Granular materials with a wide range of gradation have been used successfully as subbases for the prevention of frost damage, warping and pumping. Figs. 15, 16, 17 and 18 give the ranges of gradation and plasticity for granular materials that have given satisfactory performance as subbases under severe climatic and traffic conditions. Departures from the ranges of gradation shown for each class of material are warranted where conditions are not severe. Such departures, in some cases, will be economically desirable.

When densely graded materials are used, the subbase is constructed about 2 ft. wider than the pavement. No special provision for draining them need be made, for when properly compacted, these subbases are practically impervious.

When open-graded materials are used, some positive means of preventing the accumulation of water in the subbase should be provided. This can be done by placing these materials the full width of the grade or, if a trenched-in section is used, blind or pipe drains are placed at necessary intervals. The provision of adequate drainage adds materially to the per mile cost of subbases constructed with open-graded materials. Because of their generally higher in-place cost, their use is normally limited to the prevention of frost damage where their free-draining characteristics are desirable. When used for this purpose they should contain less than 3 per cent by weight of particles smaller than 0.02 mm.

All granular subbases, regardless of type, should be:

1. Compacted to at least 95 per cent of the material's maximum density (ASTM D698-42T or AASHO T99-49). If this is not done, faulting may occur at the joints.
2. Constructed to extend 12 in. or more beyond the edges of the pavement.

Use of Soil-Cement and Cement-Modified Soil To Produce Better Subgrades

Subgrade soils having undesirable characteristics are susceptible to improvement with portland cement. Two methods have been used:

1. Building a compacted soil-cement base course which becomes a hardened mass.
2. Modifying the soil by intermixing sufficient cement to change the soil characteristics but not to accomplish the degree of hardening which is characteristic of soil-cement.

The choice of methods will depend upon the nature of the problem, the soil, and economic considerations. Portland cement treatment of soils may be used in attaining the following objectives:

1. Control of swelling and shrinkage of high-volume-change soils.
2. Prevention of pumping and faulting on fine-grained soils.
3. Prevention of pavement damage by frost heaving.

Several projects have been constructed to determine the minimum thickness of soil-cement bases required to prevent pavement pumping and subsequent faulting. Insufficient information is available to permit definite conclusions but indications are that 4 to 6 in. will prove adequate for most conditions.

When used for these purposes there will be an increase in the bearing value of the subgrade which will permit the use of a more economical concrete pavement cross-section than would be required on the untreated subgrade. The saving in pavement cost will partly compensate for cost of treatment.

Soil-Cement — The design and construction of soil-cement is covered in detail in *Soil-Cement Laboratory Handbook* and *Soil-Cement Construction Handbook*, available free in United States and Canada from the Portland Cement Association. Cement requirements and construction specifications for soil-cement bases are the same as for soil-cement pavements.

Where soil-cement is used to prevent damage from frost heave it should be constructed to the depth and width normally required for granular materials. Construction need not be for the full depth of frost penetration because the amount of heave is not directly proportional to the depth of material frozen. Except for the most severe conditions, a combined thickness of concrete pavement and soil-cement base equal to ½ the depth of frost penetration should give satisfactory results.

Soil-cement may be used to harden granular materials whose characteristics and gradation are such that they would not provide a stable subgrade under service conditions.

A hardened base course of soil-cement has high bearing power, remains stable under freezing and thawing and wetting and drying, and is effective in preventing pavement distortion. The depth of construction should be the same as for cement-modified soil subgrades as discussed below.

Cement-Modified Soil—To accomplish this method of soil treatment, a predetermined amount of cement is intermixed with the soil and compacted at optimum moisture and to maximum density as in compacted soil-cement construction. Generally, the amount of cement required to impart improved and adequate characteristics to the soil will be substantially less than required for complete hardening.

Soils treated in this manner will have improved bearing power, reduced swelling and shrinking tendencies, and reduced tendency to water retention and consequent frost heaving.

With this method of treatment, poor soils can be modified to produce satisfactory subgrades meeting specification requirements without importation of any materials other than portland cement. Details of extended laboratory and field tests and construction operations on one experimental project were reported in *Proceedings*, Highway Research Board, 1939, Vol. 19, pages 541-551.

Generally a 6-in. depth of treatment has been found adequate except for soils having excessive volume-change characteristics, in which case greater depths are desirable. Cement contents required to control volume changes within design limits are determined by laboratory tests*.

When cement-modified soil is used to prevent damage by frost, the same test methods are used to determine cement requirements. The depth of construction should be the same as suggested for soil-cement used for the same purpose.

Specifications and construction operations for producing cement-modified soil subgrades are very similar to those used in soil-cement construction. The same mixing equipment and procedures are used. The pulverization requirement may be reduced to 70 per cent of the soil passing a No. 4 sieve when unusual pulverization problems are encountered, provided the clay lumps at the time of compaction are within 2 to 4 per cent of the optimum moisture content.

*Refer to *Soil-Cement Laboratory Handbook*, available free in U.S. and Canada from Portland Cement Association.

USE OF STEEL
In Concrete Pavements

Steel used in concrete pavements is generally in the form of tiebars, dowels and distributed steel (reinforcing). Tiebars are primarily used across longitudinal joints* where, as their name implies, they are used to tie or hold the two slabs together. Dowel bars are used across all transverse expansion joints, and under some conditions across transverse contraction joints, where the slabs have been designed under Case I (protected corners). Dowels are load transfer devices and are one of the means (see page 19) of providing the needed load transference at transverse joints. There are divergent views regarding the value of distributed steel but all highway engineers agree that it does not increase the resistance of an unbroken slab to flexural stress or add to its strength in any way. It is accepted that its sole function is to hold together the fractured faces of slabs after cracks have formed. (See page 59.)

Use and Design of Tiebars

Tiebars are deformed steel bars. They are used across the joints of concrete pavements wherever it is necessary or desirable to insure firm contact between slab faces or to prevent abutting slabs from separating.

Tiebars are required across longitudinal joints in the thin interiors of thickened-edge slabs. Their purpose in this case is to hold the faces of the slabs in intimate contact, thus insuring adequate load transference and preventing the formation of a free edge which would not have adequate load carrying capacity. When used across these thin-edged longitudinal joints, experience indicates that they should be spaced at not more than 30 in. and that the first and last tiebars in a panel should not be more than 15 in. from the joint.

Tiebars may be used across longitudinal joints in uniform thickness slabs and across longitudinal joints where the edges of the slab along such joints have been thickened. When used at such locations, tiebars are not required for structural reasons, their only function being to prevent the separation of the slabs, especially at fills or curves. When so used they may be at longer spacings and with higher working stresses than when used for structural reasons. (See page 53.)

Tiebars are not designed to act as load-transfer devices. The load transmission across a joint in which they are used is accomplished by the interlocking action ("aggregate interlock") of the rough faces of the crack that forms beneath the groove of a weakened plane (dummy groove or ribbon) type joint, or by the tongue-and-groove of a keyed type joint, see Figs. 22 and 23.

Tiebars are designed to withstand tensile stresses only. The maximum tension in the tiebars across any joint is equal to the force required to overcome friction between pavement and subgrade, from the joint in question to the nearest free

*See page 61 for the discussion of pavement joints and joint spacings.

joint* or edge. The size (diameter and length) and spacing of tiebars required across a joint may be determined by computation or may be read direct from Fig. 19.

The diameter and spacing of the tiebars are computed in the following manner:

1. The area of steel required per ft. length of joint may be computed by using Formula 10.

$$A_s = \frac{bfW}{S} \quad \text{(Formula 10)}$$

in which

A_s = area of steel required per ft. length of joint in sq.in.

b = distance between the joint in question and the nearest free joint or edge in ft.

f = coefficient of friction between pavement and subgrade (usually taken at 1.5).

W = weight per sq.ft. of slab in lb. (= 12.5 lb. per sq.ft.** × average thickness of slab in in.).

S = allowable working stress of steel in psi (see Table 9).

Example:

For tiebars across the longitudinal joint of a 10-7-10 with 2-ft. 4-in. slope thickened-edge section pavement 24 ft. wide, the area of steel required per ft. of joint would be

$$A = \frac{12 \times 1.5 \times 12.5 \times 7.29^{***}}{25{,}000\dagger} = 0.066 \text{ sq.in.}$$

2. A bar diameter is selected from Table 10 (usually ⅜ or ½ in.) whose cross-sectional area is such that when spaced at not over 30 in. it will provide the required area of steel, computed in step 1.

Example:

Table 10 shows that ⅜-in. diameter bars have a cross-sectional area of 0.11 sq.in. The spacing of these bars that will provide the required 0.066 sq.in. of steel per ft. length of joint would be

$$\text{spacing} = \frac{0.11 \times 12}{0.066} = 20.0 \text{ in.}$$

Had ½-in. diameter bars been selected, their spacing would have worked out to be $\frac{0.20 \times 12}{0.066} = 36.4$ in. The ⅜-in. bars would be used as their spac-

*The term "free joint" is applied to any type of pavement joint that has no tiebars or bonded reinforcement across it.

**Concrete taken at 150 lb. per cu.ft.

***Average thickness of a 24-ft. slab having 10-7-10 with 2-ft. 4-in. slope thickened-edge section.

†See Table 9 and its footnote.

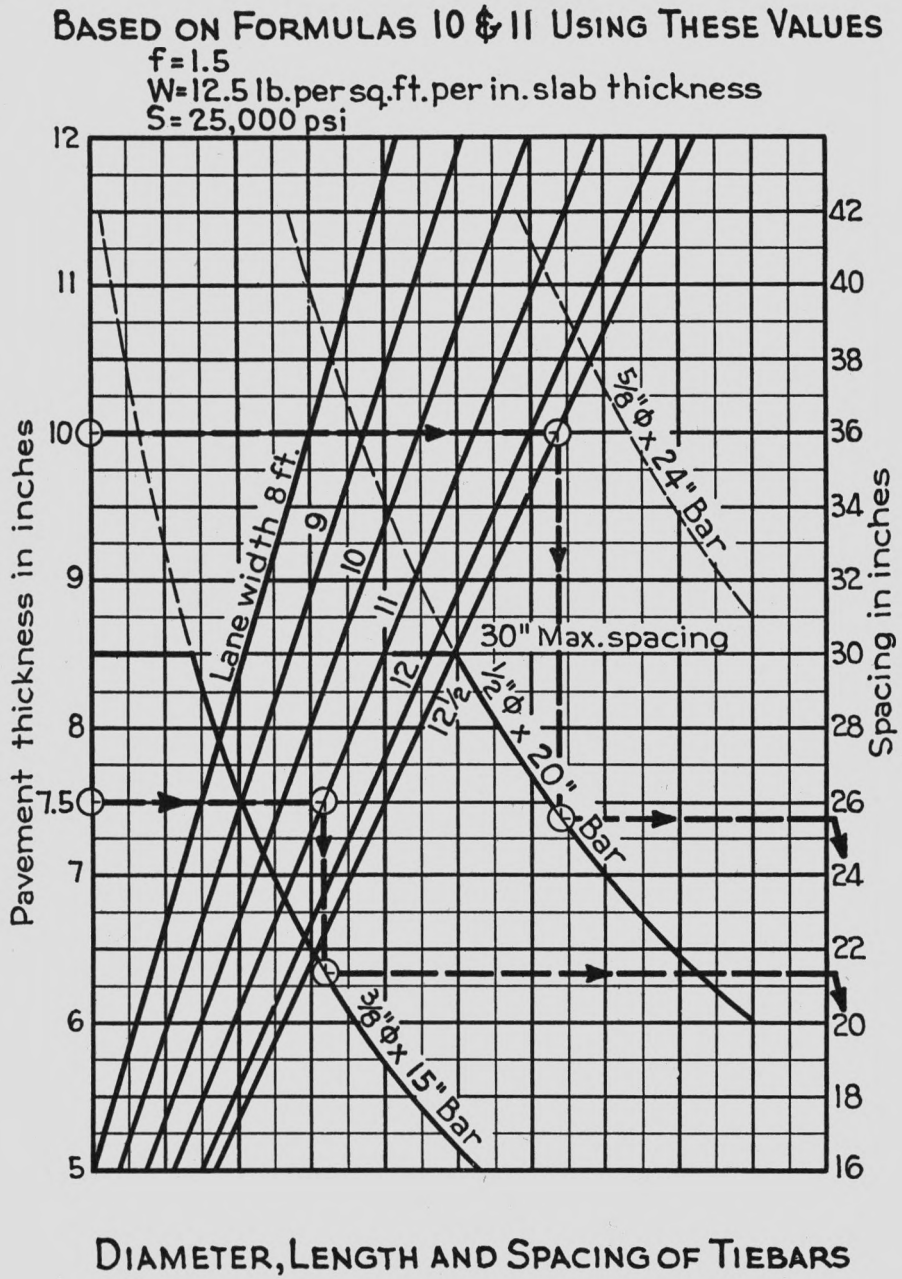


Fig. 19—Design chart for tiebars.

Table 9—Specified Yield Points (AASHO Standards) with Suggested Working Stresses

Type and grade of steel	Yield point (minimum) psi	Minimum* working stress psi
Billet and axle steel, structural grade**	33,000	16,500
Billet and axle steel, intermediate grade	40,000	20,000
Billet and axle steel, hard grade	50,000	25,000
Rail steel	50,000	25,000
Welded steel wire fabric	56,000	28,000

* There is no occasion for using working stresses lower than these for distributed steel or tiebars in concrete pavement. Many engineers make a practice of using higher working stresses in distributed steel, up to 60 per cent or even up to 80 per cent of the specified yield point of the steel. Since the consequences of an occasional failure are not so serious as in a bridge or building this practice seems to be justified.

** Specify this grade of steel for all tiebars that must be bent.

ing works out to be less than the maximum recommended spacing of 30 in.; and further experience indicates that smaller bars, with their closer spacing, are more effective since they produce the smaller concentration of stress.

The length of any tiebar should be at least twice that required to develop a bond strength equal to the working stress of the steel. Expressed as a formula, this becomes:

$$L=\frac{2\times S\times A}{350^{*}\times P} \quad \text{(Formula 11)}$$

in which

L = length of tiebar in in.

S = allowable working stress of steel in psi (use same value as employed in solving Formula 10).

A = cross-sectional area of one tiebar in sq.in.

P = perimeter of tiebar in in. (obtained from Table 10).

Example:

For the ⅜-in. diameter tiebars previously selected, their minimum length works out to be 13 in.

$$L=\frac{2\times 25{,}000\times 0.11}{350\times 1.178}=13.3 \text{ in. (use } 13+2=15 \text{ in.)}$$

Because of the relatively short length of tiebar embedded in either side of the joint (6.7 in. for a ⅜-in. bar), it is recommended that the computed length be

*The maximum working stress for bond in deformed bars (deformations meeting ASTM Specification A305) is generally taken as 0.10 of the compressive strength of the concrete, up to a maximum of 350 psi. It is permissible to use this maximum value in the design of tiebars as paving concrete should have a compressive strength in excess of 3500 psi.

increased by approximately 2 in. Thus, should the tiebar be inaccurately centered over the joint, the required minimum embedded length will be assured.

The size (diameter and length) and spacing of tiebars for use across the longitudinal joint or joints in the thin interiors of thickened-edge sections, and across keyed transverse construction joints that fall in the middle third of a normal panel-length may be read directly from Fig. 19. Values taken from this chart will correspond to those obtained from Formulas 10 and 11 when the following factors are used: $f=1.5$, $W=12.5 \times$ avg. slab thickness in in., $S=25{,}000$, and the computed length increased by approximately 2 in. to compensate for inaccuracies in placement. The method of using this design chart is indicated by the heavy dashed lines on its face.

Tiebars used across the longitudinal joints in uniform thickness slabs, or across thickened-edge longitudinal joints in thickened-edge slabs to prevent the separation of the slabs, may be designed by either of the above methods. However, the working stress of the steel used may approach its yield point. This results in a much wider spacing of the tiebars and an increase in their length. When used to prevent the separation of slab, tiebar spacings in excess of 30 in. may safely be employed. If Fig. 19 is used to design tiebars across these longitudinal joints, their length and spacing, as taken from the chart, must be increased to compensate for the higher working stress of steel permitted. These values vary directly with the working stress of the steel.

Tiebars across the longitudinal center joint of an 8-in. uniform thickness pavement, 24 ft. wide, would be designed as follows: Fig. 19 shows that ⅜-in. by 15-in. bars at 18-in. spacings are required for steel with a 25,000 psi working stress. For a working stress of 35,000 psi (yield point 40,000 psi), the ⅜-in. tiebars would be $\left(\frac{35{,}000 \times 15}{25{,}000}=21.0\right)$ 21 in. in length and spaced at $\left(\frac{35{,}000 \times 18}{25{,}000}=25.2\right)$ 24 in.

Whenever tiebars must be bent and later straightened out, as in lane-at-a-time construction, structural grade steel (see Table 9) should be used. Under

Table 10—Data on Standard Concrete Reinforcing Bars (Plain or Deformed)

Size (dia. in.)	Area (sq.in.)	Perimeter (in.)	Weight (lb./ft.)
¼ ø	.05	.785	.167
⅜ ø	.11	1.178	.376
½ ø	.20	1.571	.668
⅝ ø	.31	1.964	1.043
¾ ø	.44	2.356	1.502
1 ø	.79	3.142	2.670
1¼ ø	1.23	3.927	4.172
1½ ø	1.77	4.712	6.008
1¾ ø	2.41	5.498	8.178
2 ø	3.14	6.283	10.681

these conditions the working stress of the steel should not exceed 20,000 psi. If Fig. 19 is used to design these tiebars, the value for their length and spacing as taken from the chart should be reduced to compensate for the lower working stress of the structural grade steel. The new values are computed in the same manner as when higher working stresses are used.

Use and Design of Dowels

Dowels are mechanical load-transfer devices built as an integral part of certain transverse joints. When used they are installed across a joint in such a way as to permit the joint to open and close but to hold the slab ends on each side of the joint as nearly as possible at the same elevation. The deflection of one slab under load is resisted (through the dowel) by the other slab which, in turn, is caused to deflect and thus to carry a portion of the load imposed upon the first slab.

There are many patented or proprietary load-transfer devices on the market. Many of these have merit if properly installed at correct spacings. Smooth, round steel bars are the most commonly used type of load-transfer device, though pipe, channels, I-beams, flat bars and other shapes have been used as dowels.

Since it would be impractical to discuss the design of all the load-transfer devices that have been used, *this discussion is confined to the use and design of dowels made with smooth, round steel bars.*

Dowels are needed and used only in pavements designed under Case I—Protected Corners. They are not required when pavements are designed under Case II—Unprotected Corners.

When pavements are designed by using Formula 1 or design charts 4 and 6 (Case I), dowels must be used in the following types of transverse joints*:

1. Expansion joints.
2. Plain (butt) construction joints placed at or near the location of a transverse joint.
3. Contraction joints, where the panel lengths are greater than about 20 ft., and where the panel lengths are less than about 20 ft., if experience indicates that aggregate interlock does not provide adequate load transference.

Where dowels are required across a transverse joint they should be used throughout the full length of the joint. If dowels are placed only in the slab corners, the interior edges of the joint will have inadequate load carrying capacity. This is especially true of thin interiors of thickened-edge slabs.

If a system of dowels functioned perfectly and if the load were concentrated exactly at the slab end, 50 per cent of the load would be transferred to the ad-

*See page 61 for a discussion of joints and jointing.

jacent slab and each slab would be subject to the same unit load stress. Wheel loads are not concentrated at the slab ends, so it is not theoretically necessary to transfer the full 50 per cent of the load.

Dowel systems should be designed so that they will be capable of transferring 40 to 45 per cent* of the gross controlling wheel load to the adjacent slab when the wheel is located next to the joint and well away from the edge of the slab. Dowel systems designed in this manner, and installed the full length of the joint, will have a transfer capacity at the corners that exceeds the minimum requirements (20 per cent) for Case I design.

Design charts, Figs. 20 and 21, have been developed so that it is possible to design without calculation, dowel systems meeting the above requirements. Dowel diameter, length and spacing that will be required when the pavement thickness varies from 5 to 12 in., the subgrade support ranges from 50 to 500, and for joint openings up to 1 in., may be read directly from these charts.

Field experience shows that dowel spacings should not exceed 18 in. nor be less than 8 in.; further that the clearance between the outside dowels and the pavement edge should be within the range of 4 in. to 8 in.

Fig. 20 is to be used when designing for contraction joints where the maximum anticipated joint opening will not exceed approximately 0.15 in. Fig. 21 is to be used for expansion joints, and for contraction joints where the anticipated maximum joint opening exceeds about 0.15 in. These charts are applicable when the dowels are made up of intermediate or hard billet, intermediate or hard grade axle, or rail steel**.

A detailed discussion of the procedures, formulas, charts and tables used in developing these dowel design charts is given in Appendix II. The information given in that discussion can be used to check values taken from Figs. 20 and 21 or it may be used to design dowel systems that cannot be obtained directly from the charts.

A study of the discussion in Appendix II will reveal that many factors must be taken into account when designing a dowel system. The load-transfer capacity of a single dowel is governed by its capacity in bending and bearing. These are influenced by the width of the joint opening, tensile or bending stress in the steel, diameter of the dowel, length of dowel embedded in the concrete and the bearing stress on the concrete. The load-transfer capacity of a series of dowels is dependent not only upon the capacity of the individual dowels and their spacing but also upon the pavement thickness, the *k* value of the subgrade, the center-to-center spacing of the wheels on the heavier axles, and the location of the wheels with respect to the edge of the pavement. The interrelationship between these factors is such that the curves making up the design charts, Figs. 20 and 21, do not follow an absolutely regular pattern.

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*F. S. Snow, Airport Paper No. 12, *Institution of Civil Engineers*, Session 1949-50, page 21, reports the results of full-scale field tests which substantiate this value.

**Dowels should be made up only of the grades of steel indicated.

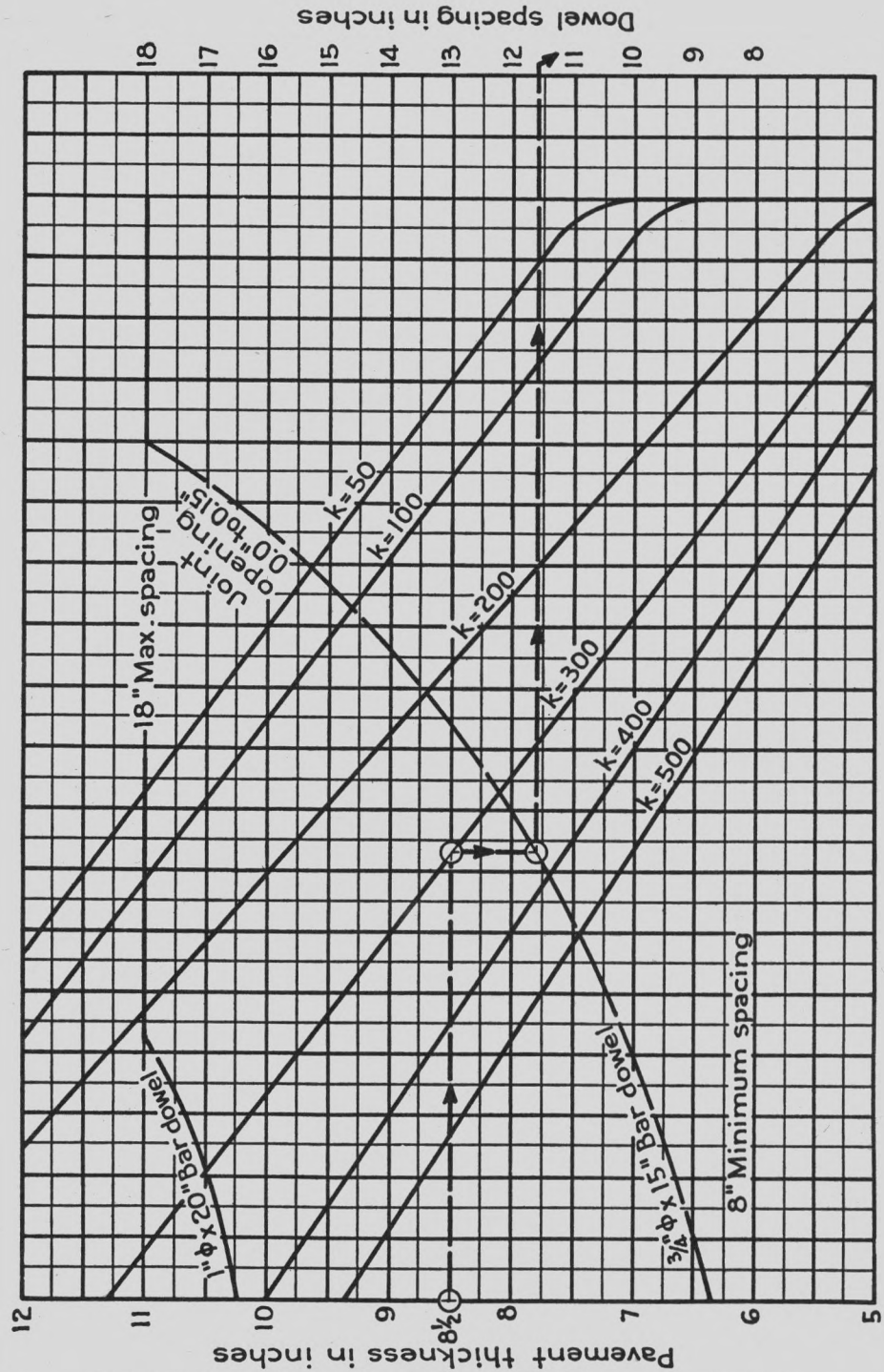


Fig. 20—Design chart for smooth round steel dowels used across contraction joints.

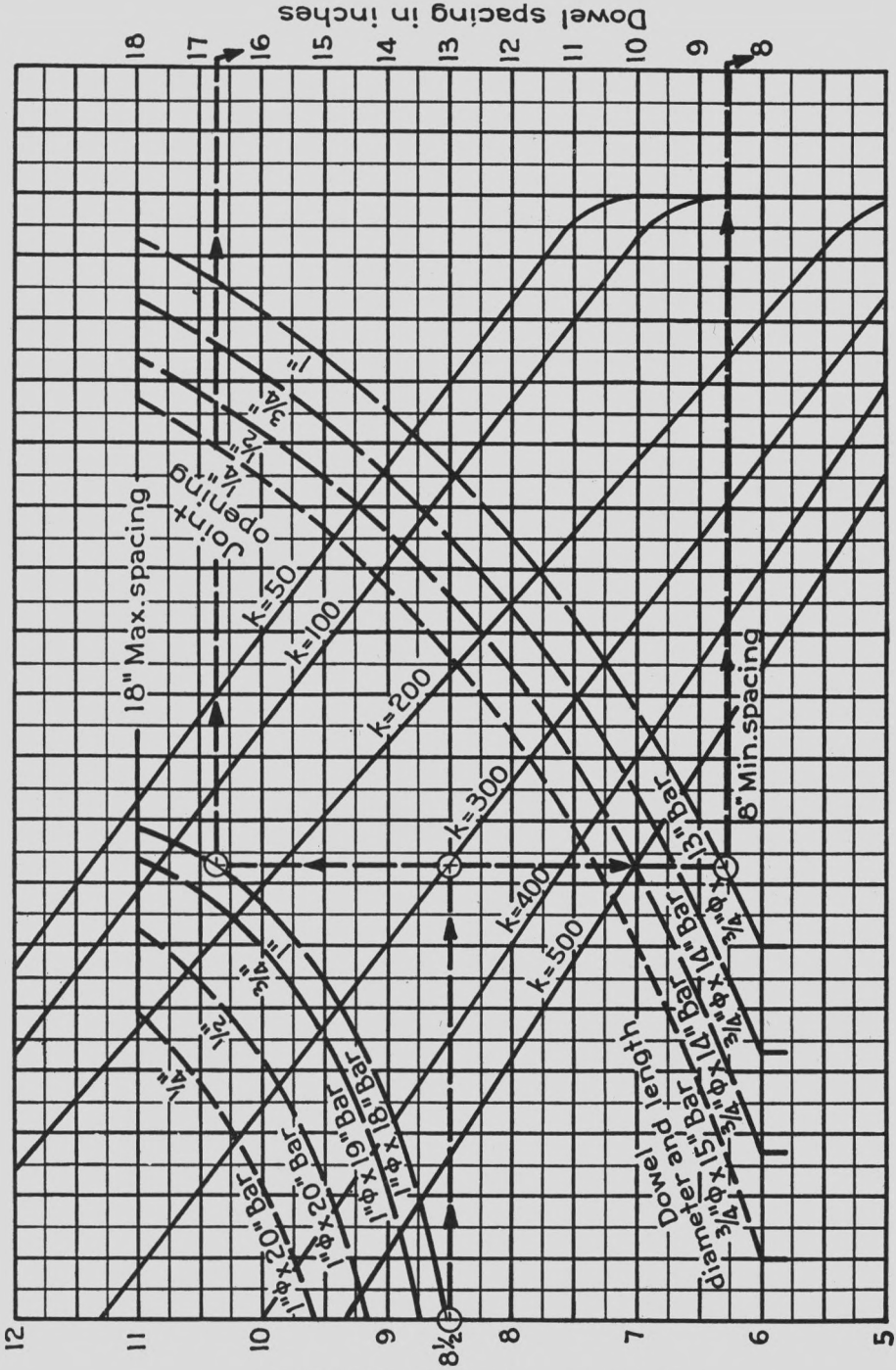


Fig. 21—Design chart for smooth round steel dowels used across expansion joints.

When designing dowel systems by means of the charts it is necessary to know the pavement thickness (for thickened-edge sections use the value of *d* rather than the average thickness), the *k* value of the subgrade, the type of joint, and the width of the joint opening that is to be spanned.

The width of the joint opening need not be considered when designing for contraction joints if the panel lengths are less than about 30 ft. and expansion joints are spaced more than about 250 ft. When the jointing of the pavement meets these conditions the maximum anticipated joint opening will not exceed the maximum joint opening (0.15 in.) for which Fig. 20 is applicable. When the jointing scheme does not meet these conditions the maximum opening of the contraction joints will generally be in the range of 0.15 to 0.35 in., a ¼-in. joint opening should be assumed, and Fig. 21 used.

Example problems will serve to show how dowel systems are designed by means of design charts, Figs. 20 and 21.

Pavement: A thickened-edge slab, 24 ft. wide, having a 10-7-10 with 2-ft. 4-in. slope* cross-section, dummy groove contraction joints spaced at 20 ft., and 1-in. expansion joints used only at the outside end of the approach slabs of structures and intersections.

Example (contraction joints)—In this pavement, the designer may depend upon aggregate interlock to supply adequate load transference at all contraction joints except for the 6 to 10 joints adjacent to expansion joints. (See page 65.) The following example is for the design of dowels at only these 6 to 10 joints.

Since the slab has a thickened-edge cross-section, its effective *d* is used (rather than its average thickness) where pavement thickness is indicated. The effective *d* for the slab is 8.24 in., see Tables 5 or 8. The *k* value of the subgrade is 100. Fig. 20 will be used since the panel length (joint spacing) is less than 30 ft. and the expansion joint spacing exceeds 250 ft.

Enter the chart at the left-hand edge at a position corresponding to the pavement thickness (8.24); proceed horizontally until the proper *k* (100) line is intersected; thence vertically (upward, this case) until the ¾-in.x15-in. bar dowel curve is intersected (on the horizontal portion in this case); thence horizontally to the right-hand edge where the dowel spacing (18 in.) is read. This procedure is illustrated by the dashed line on the face of the chart.

The dowel system for use across these contraction joints would consist of ¾-in.x15-in. smooth, round steel bars spaced at not over 18 in. If an actual spacing of 16 in. is used, 18 bars will be required and there will be an 8-in. clearance** between the outside dowels and the edge of the pavement (17×16 in.+2×8 in.=288 in. or 24 ft.).

Example (expansion joints)—Pavement thickness (use *d*)=8.24, subgrade *k*=100, joint opening=1 in. Fig. 21 will be used since these are expansion joints.

*Designed in Table 7.

**See page 55.

The chart is read in a manner similar to that used to read Fig. 20. Enter the chart at the left-hand edge at a position corresponding to the pavement thickness (8.24); proceed horizontally until the proper k (100) line is intersected; thence vertically (upward, this case) until the proper joint opening (1 in.) curve is intersected, at the lower end of this curve the dowel size (¾-in.x13-in.) is read; thence horizontally to the right hand edge where the dowel spacing (13½ in.) is read. This procedure is illustrated on the face of the chart.

The dowel system for use across the 1-in. expansion joints would consist of ¾-in.x13-in. smooth, round steel bars spaced at not over 13½ in. If an actual spacing of 13 in. is used, 22 bars will be required and there will be a 7½-in. clearance between the outside dowels and the edge of the pavement.

Use and Design of Distributed Steel

Distributed steel used in highway pavements is in the form of welded wire fabric or bar mats. Its principal function is to hold together the fractured faces of slabs after cracks have formed. Adequate load transmission across the crack is thus assured (by the interlocking action of the rough faces of the crack) and the infiltration of incompressible material into the crack is prevented. It does not increase the flexural strength of an unbroken slab when used in the quantities which are considered to be within the range of practical economics. Where pavements are jointed in a manner that will control cracking, distributed steel has no function.

The jointing of pavements that will control cracking is discussed on pages 64 and 65. When joint spacings (panel lengths) in excess of those given on page 65 are used, distributed steel serves to prevent the opening of intermediate cracks which may be expected to develop in the central portion of each slab. The amount of steel needed to keep these cracks from opening may be calculated as explained in following paragraphs.

Like tiebars, distributed steel is designed to withstand tensile stresses only. The maximum tension in the steel members across a crack is equal to the force required to overcome friction between pavement and subgrade from the crack to the nearest free joint* or edge. This force will be the greatest when the crack occurs at the middle of a slab. Distributed steel is designed to withstand the tension stresses when a crack develops at this location. For practical reasons the steel is usually made the same weight throughout the length of short slabs, but on the longer slabs (over 50 ft.) considerable saving can be realized by using lighter steel for portions of the slab near its ends. This would be practical only where the savings which might be effected by varying the weight of the steel would be great enough to pay for the additional labor and inspection which would be required to handle the different weights, fabric or mats and to make sure that they are placed in the correct locations.

*See footnote *, page 50.

The amount of longitudinal and transverse steel required per 1-ft. width or length of slab is computed by the following formula:

$$A_s = \frac{LfW}{2S} \qquad \text{(Formula 12)}$$

in which

A_s = area of steel in sq.in. required per ft. width or length of slab.

L = distance in ft. between free transverse joints when the equation is used to calculate longitudinal steel or between free longitudinal joints or edges when figuring transverse steel.

f = coefficient of friction between slab and subgrade.

W = weight in lb. of 1 sq.ft. of slab.

S = allowable working stress, psi, in the steel.

When computing the value of W for use in this formula, concrete is generally assumed to weigh 150 lb. per cu.ft. (12.5 lb. per sq.ft. per in. of slab thickness). The value of f may range from 1 to 2, but 1.5 is the most commonly used value. S is usually taken as approximately half the yield point of steel. The specified minimum yield point and the corresponding minimum working stress for the various grades of steel used in bar mats, welded wire fabric and tiebars are given in Table 9.

Experience indicates that small bars or mesh are more effective than the same area of larger bars because they can be distributed more uniformly in the slab. This fact and the higher working stress that may be used with welded wire fabric must be balanced against the higher price per pound of the welded wire fabric.

The first step in the design of distributed steel is to determine the amount of longitudinal and transverse steel required for the type of reinforcement (welded wire fabric or bar mat) used. With these values established it is then a matter of determining the size and spacing of the members so that they will provide the required cross-sectional area of steel per foot of pavement in each direction.

Since distributed steel is not intended to act in flexure, its position within the slab is not important except that it should be adequately protected from corrosion. It is usually placed about 2 in. below the surface. This is accomplished by striking off the concrete 2 in. below the final surface, placing the mats or mesh on this surface, and then placing the remainder of the concrete.

Distributed steel is sometimes continued across the longitudinal joint to serve as tiebars. This is often done with the dummy groove or ribbon type joint but it is not practical to do so with the deformed metal plate type of longitudinal joint.

JOINTING of Concrete Pavement

With the exception of construction joints, whose purpose is self-evident, joints are used in concrete pavements to reduce stress caused by changes in volume of the concrete (see page 13). Longitudinal and transverse joints are installed in concrete pavements to keep these stresses within safe limits and thus prevent the formation of irregular cracks.

All joints in concrete pavements, regardless of type, are normally installed at the time the pavement is constructed*. They are made by forming a groove in the fresh concrete or by placing within the concrete, strips of metallic or non-metallic material in such a manner as to form a plane of weakness or a complete plane of separation.

Longitudinal Joints

Longitudinal joints are used to control longitudinal cracking. The more common methods of forming longitudinal joints are shown in Fig. 22. Of these, the dummy groove and the ribbon or premolded strip types are most widely used when two or more lanes are constructed in one operation. They are least expensive to construct and function properly when their depth is equal to ¼ to ⅛ the pavement thickness.

A longitudinal joint is placed at the centerline of two-lane pavements up to 25 ft. in width. Longitudinal joints in multiple-lane pavements are spaced so that they coincide with the lane markings; this spaces them at 8- to 12½-ft. intervals.

In multiple-lane pavements not more than 4 lanes should be tied together. When more lanes are tied together, longitudinal cracks may develop at the ends of the tiebars. Pavements more than 4 lanes in width should be provided with one or more free (without tiebars or bonded reinforcement) longitudinal joints. The pavement edges at these free joints must be thickened if the pavement has been designed as a thickened-edge slab. This is necessary because the pavement edges at these free joints will receive little or no support from the abutting slab.

Where lane-at-a-time construction is used and in multiple-lane pavement too wide to be built full width, the construction joint serves as a longitudinal joint. All longitudinal construction joints should be keyed, regardless of whether they are tied or free.

Transverse Joints

Transverse joints are used to control transverse cracking. They fall in two general classes—expansion joints and contraction joints. Expansion joints relieve compressive stresses that develop when a pavement expands by providing

*Some projects have been constructed where contraction joints have been sawed in the concrete following its placement. There are not yet sufficient data to permit definite conclusions as to the practicability and economics of this scheme.

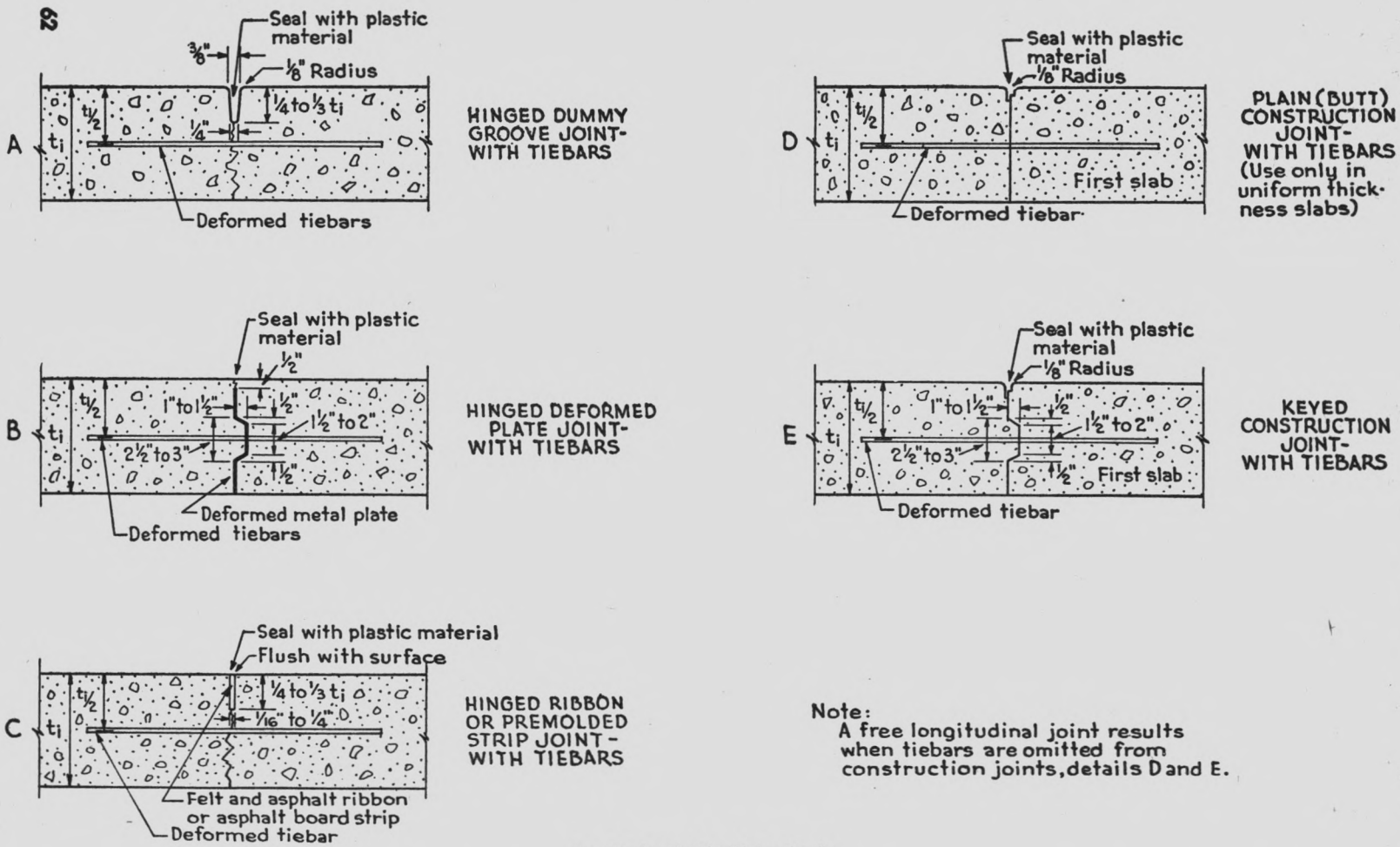
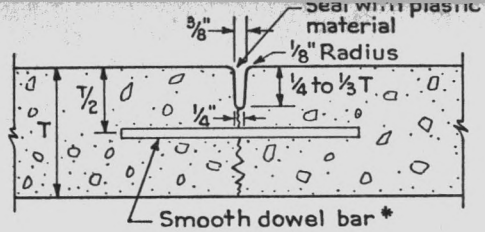
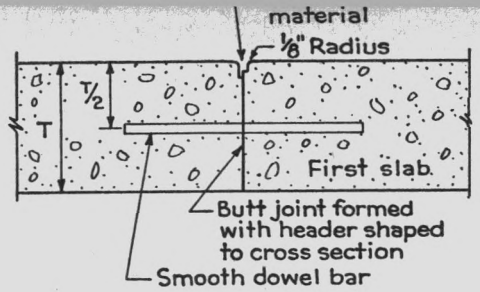


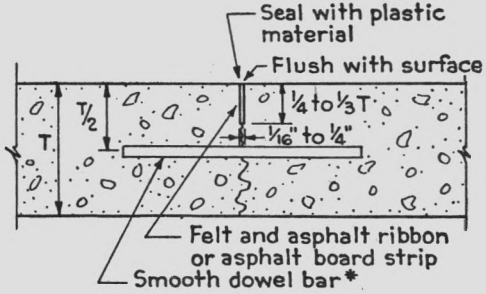
Fig. 22—Longitudinal joint details.



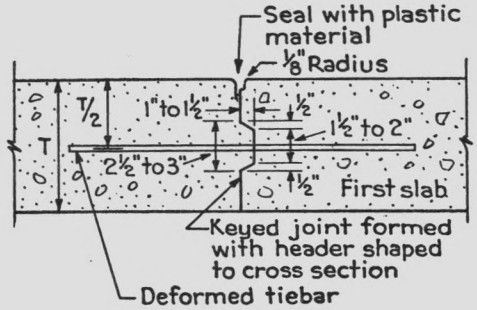
DUMMY GROOVE CONTRACTION JOINT



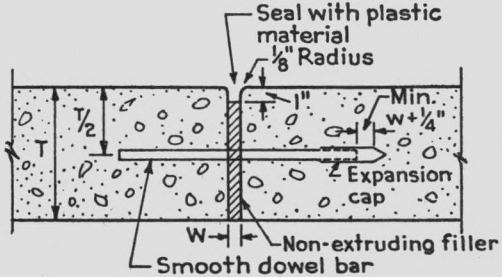
CONSTRUCTION JOINT- WITH DOWELS (Place only at location of transverse joint)



RIBBON OR PREMOLDED STRIP CONTRACTION JOINT



KEYED CONSTRUCTION JOINT- WITH TIEBAR (Place only in middle third of normal joint interval)



EXPANSION JOINT

Note:-
All dowel bars across transverse joints shall be painted and oiled to prevent bond with concrete. Dowel bars across expansion joints shall be provided with expansion caps.

*Required only under certain conditions. See text

Fig. 23—Transverse joint details.

space into which slab ends may move. Contraction joints relieve the tensile stresses that develop when a reduction in volume of concrete takes place. Details covering the more common types of transverse joints are shown in Fig. 23.

Expansion Joints—Expansion joints are formed as indicated in Fig. 23 by embedding in the concrete a vertical layer of nonextruding compressible material so shaped that there is a complete separation between the abutting slabs. The concrete above the filler material must be removed, as shown in Fig. 23, and the exposed edges of the joint rounded with an edging tool. This space above the joint filler is later sealed with a plastic material to prevent the infiltration of foreign matter.

The dowels used across expansion joints must be equipped with tubular metallic expansion caps, shown in Fig. 23. These close-fitting caps provide space into which the end of the dowel may move when the joint closes. The design of dowels to be used across expansion joints is discussed on page 54.

Laboratory tests, field data obtained from experimental projects * and studies that have been made on pavements in service show that, except at structures and intersections, *no expansion space (expansion joints) need be built into concrete pavements when:*

1. the pavement is constructed of materials that have normal expansion characteristics;
2. the pavement is constructed during those periods of the year when normal construction temperatures prevail;
3. the pavement is divided into relatively short panels by contraction joints so spaced as to prevent the formation of intermediate cracks; and
4. the contraction joints are properly maintained to prevent the infiltration of relatively incompressible material, such as soil fines.

Concrete pavements built during cold weather, or constructed of materials having high coefficients of expansion should be provided with ¾- to 1-in. expansion joints at 600- to 800-ft. intervals. Expansion joints should be used where pavements join structures, such as bridges and railroad tracks, or at the intersection with other pavements. The width of these expansion joints need not be excessive. They usually range in width from ¾ to 1½ in., 1-in. width being the most common.

Contraction Joints—Contraction joints are formed by using one of the methods detailed in Fig. 23. The types shown, the dummy groove and the ribbon or premolded strip contraction joints, are the most widely used. They are inexpensive to construct and will function properly when their depth is equal to ¼ to ⅓ the pavement thickness.

Contraction joints in *unreinforced* concrete pavement should be spaced at intervals which will control cracking. Local experience is the best guide as to joint interval (panel length). Should no experience records be available, the following tabulation may be used as a guide when selecting a spacing for con-

*See footnote *, page 16.

traction joints. The joint spacings given have satisfactorily controlled cracking in pavements on average subgrade soils and under most climatic conditions.

Kind of coarse aggregate	Joint spacing ft.
Granite	25
Limestone	20
Flinty limestone	20
Gravel—calcareous	20
—siliceous	15
—less than ¾ in. in size	15
Slag	15

If the above recommended spacings are followed and the joints are properly designed, cracking will be reduced to a point where the few cracks that may occur will be of such little significance they will require no special attention.

Dowels are not required across the contraction joints in unreinforced concrete pavements constructed without expansion if the contraction joints are spaced at intervals which will control cracking. This is the case since the slab lengths will be short enough so that even under conditions of maximum contraction only a very small joint opening will develop. The interlocking of the aggregate particles at the faces of the joint will provide a substantial and adequate amount of load transference, thus making the use of dowels unnecessary.

Dowels are required across all contraction joints when the joint spacing exceeds about 20 ft., or when expansion joints are spaced at less than about 500 ft. In either instance, the joint opening that can be expected during those seasons of the year when the slab has a minimum volume will be great enough so that aggregate interlock cannot be relied upon to supply adequate load transference. The design of dowels to be used across contraction joints is discussed on page 54.

Dowels are required across only the first 6 to 10 contraction joints* either side of an expansion joint when the expansion joint spacing is greater than about 500 ft. and the contraction joint spacing does not exceed about 20 ft. The first 6 to 10 contraction joints out from expansion joints adjoining structures must also be doweled, even though no expansion joints are used within the remainder of the pavement. The doweling of these contraction joints is necessary since they tend to open much wider than do those farther removed from an expansion joint. The amount of joint opening tends to decrease inversely with the distance away from the expansion joint and at 6 to 10 joints back from an expansion joint is essentially the same as in a pavement without expansion joints.

Contraction joint spacings or panel lengths in pavements with wire fabric or bar mats should be established only after evaluating the following factors—(1) the weight and in-place cost of the distributed steel that will be required, (2) the

*Where expansion joint filler is wood or where the climate is so equable that volume changes are at a minimum, the number of joints requiring dowels may be reduced. Under most favorable combinations of circumstances, dowelled contraction joints may be eliminated or the number reduced to 1 or 2 each way from the expansion joint.

number and in-place cost of the doweled contraction joints that will be required, and (3) the maximum seasonal changes in joint opening that can be expected.

The joint spacing that produces the lowest total in-place cost of the distributed steel and doweled joints would, of course, be the most economical. Under most conditions it will be found that this joint spacing will be in the range of 30 to 35 ft. It is exceedingly difficult to maintain an adequate seal at a joint where the maximum seasonal movement exceeds about 0.20 in. Observations made on experimental jointing projects * show that panel lengths in excess of about 40 ft. will generally produce seasonal variations in joint opening that are in excess of 0.20 in. When this fact is considered, along with the economics of the problem, it is evident why contraction joint spacings in excess of about 40 ft. are seldom justified when reinforced concrete pavement is used.

All contraction joints in reinforced concrete pavement with panel lengths over about 20 ft. should be doweled. Because of the longer panel lengths, the maximum joint openings that occur will be such that aggregate interlock cannot be relied upon to produce the needed load-transference across the joint.

Contraction joints spaced at 30 ft. or more should be of a type that provides a sizable groove or reservoir to hold joint sealing material. This is essential for proper sealing as the amount of seasonal movement at these joints may be many times greater than that which takes place in joints at shorter spacings.

Hinged Joints

Joints with bonded steel are used to provide relief for flexural stresses caused by curling of the slab. Longitudinal joints with tiebars fall in this class of joint. Hinged transverse joints have little use where spacing of contraction and/or expansion joints is such as to control cracking.

Seal Materials

The groove or crack above all longitudinal and transverse joints should be sealed with plastic material after the concrete hardens and before opening the pavement to traffic (see Figs. 22 and 23). If bituminous material is used for sealing the joints, it should be as soft as can be used without flowing from the joint in warm weather. Mineral fines, such as diatomaceous earth, when mixed with bituminous material will permit the use of higher penetration bitumens which will better retain their sealing properties at low temperatures. Certain rubber-bituminous sealing compounds have given good results where they have been properly handled during placement.

*See footnote * page 16.

APPENDIX I

Measuring the Supporting Value of Subgrades

Plate Bearing Test

Preparation of the Test Area—The manner in which the test area is prepared will depend upon the nature of the construction. The construction may consist of grading and paving a new location, or paving a previously graded roadway which has been in service. In the former, the test may be made during the preliminary soil survey or after some rough grading has been completed.

It is essential, whenever practical, to conduct the test on representative soils at densities and water contents which approximate service conditions under the pavements*. To do this during the preliminary survey an area may be stripped and a small test embankment about 30 in. high can be constructed to conditions which will approximate those expected in service. Some soils, especially the more pervious silts, may be compacted and ponded. Best results on clays may be obtained by compacting them at the water contents and to the densities which they are expected to attain in service.

Testing Equipment—Several different methods of applying the load and measuring deflections have been used. A typical set of test equipment consists of a 30-ton hydraulic jack equipped with pressure gage and properly calibrated, two dial-type gages sensitive to 0.001 in., a 4-in. channel 12 to 15 ft. long for mounting the dials, and a rigid steel bearing plate. For practical reasons a plate 30 in. in diameter generally has been used. For smaller plates the size of loaded area has considerable influence on the penetration and on the resulting value of k. Tests with plates of about 30-in. diameter and larger show little influence of size of loaded area. Larger plates require excessive loads. A ball joint should be provided between the test load and jack or between the jack and the test plate to prevent eccentric loading of the plate. A wide-flanged beam of 12- to 15-in. size and 18 to 20 ft. long can be used in conjunction with loaded trucks to provide the reaction for applying the load through the jack to the bearing plate. The reaction apparatus may consist of a suitable weight of material with the necessary beams for supporting it over the test area, or it may consist of screw anchors or deadmen together with beams required to enable the jack to act against the anchors (see Figs. 24 and 25).

Application of the Load—The equipment is assembled so the bearing plate is at least 8 ft. from the nearest support of the dead load or anchor. After the area has been leveled off a thin layer (less than ¼ in.) of dry, fine sand may be used to obtain good seating of the bearing plate.

*The U.S. Engineer Department has devised a laboratory method to adjust for field conditions when it is not practical to conduct plate bearing tests at the expected soil condition. See *Proceedings*, Highway Research Board, Vol. 22, 1942, pages 170-171; or *Proceedings*, American Society of Civil Engineers, Vol. 70, No. 1, 1944, pages 32-33.

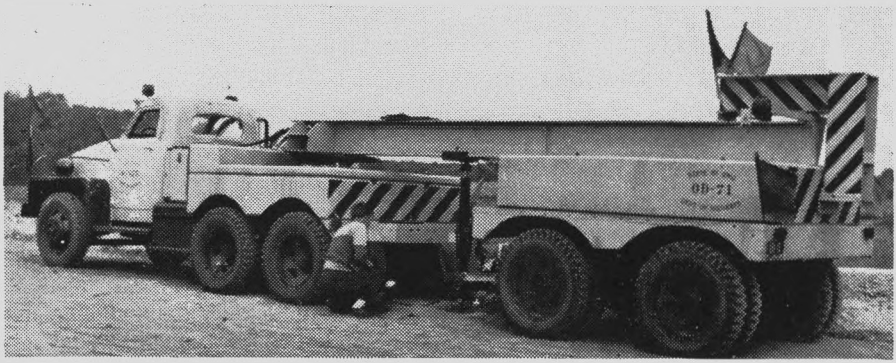
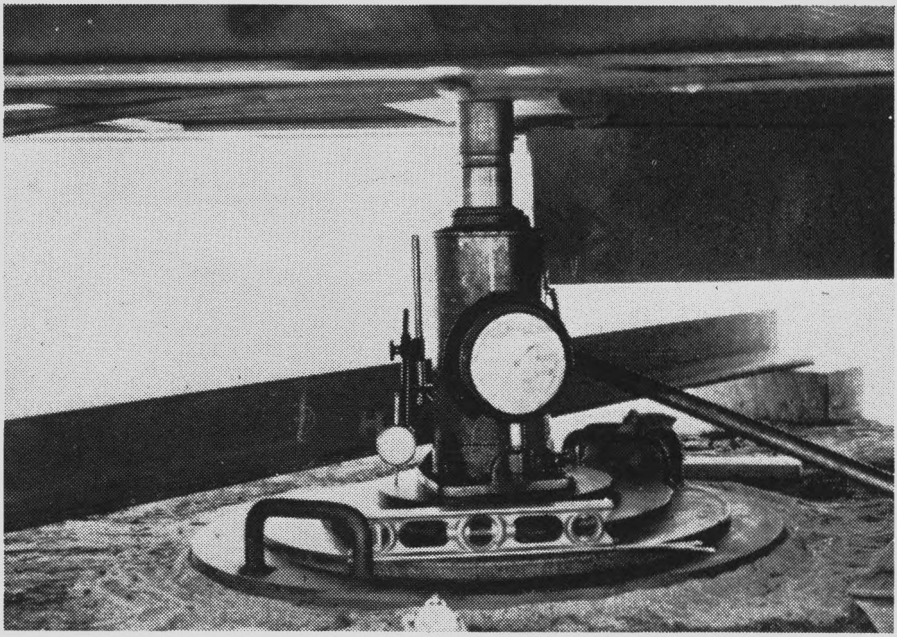


Fig. 24—Special truck and trailer used by Ohio Department of Highways as the reaction for plate bearing tests made on subgrade.

A load of 5 psi is applied and released to seat the plate before beginning the test, except for very soft soils for which a smaller seating load may be necessary. The test load is then applied in 5-psi increments, allowing practically complete deflection for each increment before another increment of load is applied. A good rule is to allow the deflection under each load increment to continue until the increase in deflection in 5 minutes is less than 3 per cent of the increase in

Fig. 25—Typical setup for plate bearing test. Note the sandbedding course between the plate and the subgrade.



deflection for that load. For soft clay soils it may be advantageous to plot time-deflection curves to aid in determining when practically complete deflection is obtained.

Evaluating Test Results—The load-deflection data may be plotted in the form of a curve shown in Fig. 26. The modulus of subgrade reaction k is the ratio

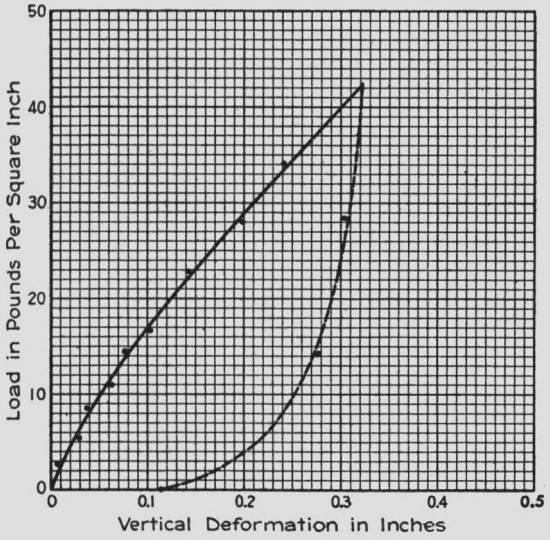


Fig. 26—Typical load-deflection curve plotted from data obtained during a plate bearing test.

of load in psi to the displacement of the bearing plate in inches. For example if the load-deflection curve shows that a load of 10 psi results in a deflection of 0.05 in., $k = \frac{10}{0.05}$ or 200 psi per in. The displacement or deflection of the bearing plate used in determining k should not exceed the deflection of pavement slabs under expected wheel loads. The load-deflection ratio at a deflection of 0.05 in. is generally used in determining k unless the load-deflection curve indicates that the load required to produce this deflection is beyond the capacity of the soil, in which case a lower deflection is selected.

Determining *k* by Loading Existing Pavement

A pavement slab, when loaded to determine the supporting value of a subgrade soil, will distribute the load over a greater area than the area of a 30-in. plate. Therefore, a jack of greater capacity and greater loads will be required.

Both the area of slab deflected and the deflection must be known to determine the volume of displacement under various loads. Several gages are required for this purpose. When using this method, k is the ratio of total load in lb. to total volume of displacement in cu.in.

APPENDIX II

Details of Dowel* Design and Development of Dowel Design Charts

Dowel systems (diameter, length and spacing of dowels) should be designed so that they will be capable of transferring 40 to 45 per cent** of the gross controlling wheel load to the adjacent slab when the wheel is *located* next to the joint and well away from the edge of the slab. Systems designed in this manner, and installed the full length of the joint, will have transfer capacities at the slab corners that exceed the minimum requirements (20 per cent) for Case I design.

The first step in the design of a system of dowels is to determine the load-transfer capacity of a single dowel. This is done by computing its capacity in shear, bending and bearing. These are influenced by the width of the joint opening, allowable tensile and shear stress in the steel, diameter of the dowel, length of dowel embedded in the concrete, and the allowable bearing stress on the concrete.

The load-transfer capacity of a single steel bar dowel is given by the following formulas***.

Capacity in shear (lb.) $= P_s = 0.785\, d^2 f'_s$ (Formula 13)

Capacity in bending (lb.) $= P_b = \dfrac{2\, d^3 f_s}{r + 8.8Z}$ (Formula 14)

Capacity in bearing of concrete (lb.) =

$$P_c = \frac{f_c r^2 d}{12.5\,(r + 1.5Z)} \quad \text{(Formula 15)}$$

in which

d = diameter of dowel in in.
f'_s = shearing stress in steel (psi).
f_s = tensile or bending stress in steel (psi).
f_c = bearing stress on concrete (psi).
r = length of dowel embedded in concrete (in.).
Z = width of joint opening (in.).

The actual length of each dowel is $r + Z$.

Usual allowable working stresses are 15,000 psi for shear (f'_s), 30,000 psi for tension (f_s), and 1800 psi for bearing (f_c). Inspection of formulas 14 and 15 indicates that the capacity of a dowel in bearing on the concrete increases with its

*This discussion is confined to the design of smooth, round steel bar or pipe.

**See page 55 for explanation of this value.

***From "Design of Joints in Concrete Pavements" by R. D. Bradbury, *Proceedings*, Highway Research Board, Vol. 12, 1932, pages 105–136.

length, while its capacity in bending decreases with its length. For equal capacity in bending and bearing the length is

$$r=d\sqrt{\frac{25f_s}{f_c}}\sqrt{\frac{r+1.5Z}{r+8.8Z}} \quad \text{(Formula 16)}$$

Steel pipe dowels offer greater resistance to bending and greater bearing area between dowel and concrete per lb. of steel used than do bar dowels. Corresponding formulas for pipe dowels are as follows:

$$\text{Capacity in shear (lb.)}=P_{ps}=0.785\,(d^2-d_1^2)\,f'_s \quad \text{(Formula 17)}$$

$$\text{Capacity in bending (lb.)}=P_{pb}=\frac{2f_s(d^4-d_1^4)}{d(r+8.8Z)} \quad \text{(Formula 18)}$$

in which

d_1 = inside diameter of pipe.

d = outside diameter of pipe.

The capacity of pipe dowels in bearing on concrete is same as given for bar dowels in formula 15. For equal capacity in bending and bearing the length is

$$r=\sqrt{\frac{d^4-d_1^4}{d^2}}\sqrt{\frac{25f_s}{f_c}}\sqrt{\frac{r+1.5Z}{r+8.8Z}} \quad \text{(Formula 19)}$$

The load-transfer capacity of any bar or pipe dowel is governed by its capacity in bending and bearing, since for dowel diameters and lengths employed in highway pavements these capacities are of lesser magnitude than a dowel's capacity in shear.

Formulas 16 and 19 are not adapted to direct solution for value of r, but for bar and pipe dowels of sizes suitable for use in street and highway pavement the lengths which will have equal capacity in bearing and bending can be read directly from Fig. 27. Similarly the load-transfer capacity of individual dowels of the same sizes may be read from Fig. 28. Values in the latter chart are for dowels of lengths given in Fig. 27 so that they have equal capacity in bending and bearing. Figs. 27 and 28 are prepared for $f_s=30{,}000$ psi and $f_c=1800$ psi.

The second step in the design of a dowel system is to determine the load-transfer capacity of the series of uniformly spaced dowels that make up the system. The load-transfer capacity of a series of dowels is influenced by their spacing, their position with respect to a wheel load, the load-transfer capacity of the individual dowels, pavement thickness, subgrade modulus k, and the center-to-center spacing of the dual wheels of the axle carrying the gross controlling wheel loads.

The dowel spacing and position with respect to a wheel load enters the problem in the following manner. The dowel directly under the point of loading (center of wheel load) is the most effective, and the effectiveness of the other dowels in the series decreases to zero at a distance of $1.8l$* from point of load**.

*l, the radius of relative stiffness, as defined by Formula 3, page 17, values are given in Table 11.

**B. F. Friberg, "Design of Dowels in Transverse Joints of Concrete Pavements", American Society of Civil Engineers *Proceedings*, 1938, Vol. 64, No. 9, pages 1809-1828.

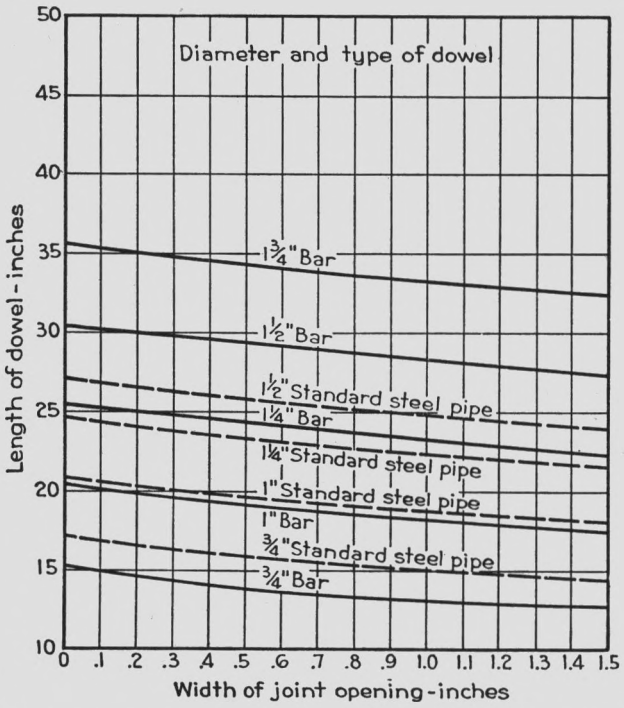


Fig. 27—Length of dowels that are balanced in bearing and bending.

The dowel directly under the wheel load will transfer its full capacity, as taken from Fig. 28. When computing the load transferred by the other dowels in the series, it is sufficiently accurate to use a straight line relation, varying from

Table 11—Values of l in In. for $E = 4{,}000{,}000$ Psi and $\mu = 0.15$

Radius of relative stiffness $= l = \sqrt[4]{\frac{Ed^3}{12(1-\mu^2)k}} = 24.1652\sqrt[4]{\frac{d^3}{k}}$

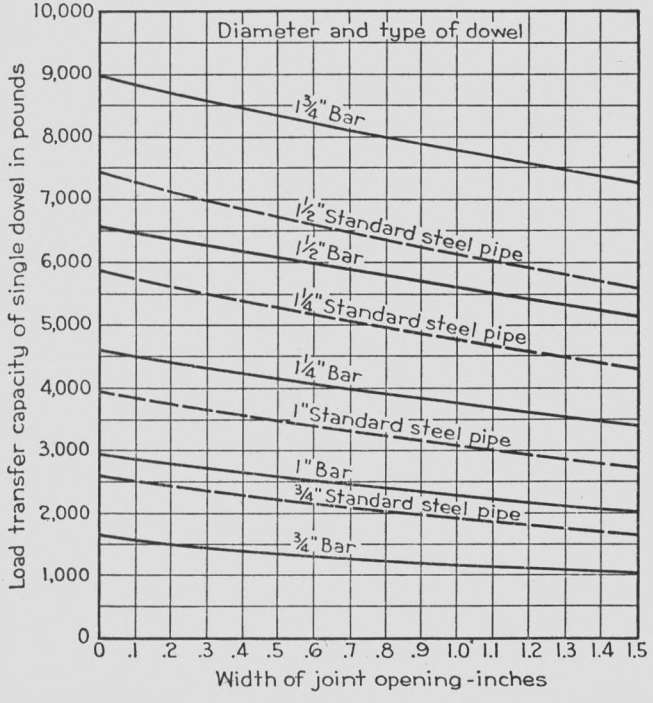
d in in.	$k = 50$	$k = 100$	$k = 150$	$k = 200$	$k = 250$	$k = 300$	$k = 350$	$k = 400$	$k = 500$
4	25.70	21.61	19.53	18.18	17.19	16.42	15.80	15.28	14.45
4.5	28.08	23.61	21.33	19.85	18.78	17.94	17.26	16.69	15.79
5	30.39	25.55	23.09	21.49	20.32	19.42	18.68	18.07	17.09
5.5	32.64	27.44	24.80	23.08	21.83	20.85	20.07	19.41	18.35
6	34.84	29.30	26.47	24.63	23.30	22.26	21.42	20.72	19.59
6.5	36.99	31.11	28.11	26.16	24.74	23.64	22.74	22.00	20.80
7	39.11	32.89	29.72	27.65	26.15	24.99	24.04	23.25	21.99
7.5	41.19	34.63	31.29	29.12	27.54	26.32	25.32	24.49	23.16
8	43.23	36.35	32.85	30.57	28.91	27.62	26.58	25.70	24.31
8.5	45.24	38.04	34.37	31.99	30.25	28.91	27.81	26.90	25.44
9	47.22	39.71	35.88	33.39	31.58	30.17	29.03	28.08	26.55
9.5	49.17	41.35	37.36	34.77	32.89	31.42	30.23	29.24	27.65
10	51.10	42.97	38.83	36.14	34.17	32.65	31.42	30.39	28.74
10.5	53.01	44.57	40.28	37.48	35.45	33.87	32.59	31.52	29.81
11	54.89	46.16	41.71	38.81	36.71	35.07	33.75	32.64	30.87
11.5	56.75	47.72	43.12	40.13	37.95	36.26	34.89	33.74	31.91
12	58.59	49.27	44.52	41.43	39.18	37.44	36.02	34.84	32.95

maximum directly under the center of the wheel load to zero at distances of 1.8*l* either side of the load. The transfer capacity of a system of dowels is the summation of the loads transferred by the individual dowels that are within a distance of 1.8*l* of the point of load.

The principle stated above was used when preparing Fig. 29. This chart permits the direct determination of the "capacity factor" of a system of dowels where the center-to-center spacing of the dual wheels on an axle equals or exceeds 1.8*l*. This capacity factor when multiplied by the load-transfer capacity of a single dowel (obtained from Fig. 28) gives the transfer capacity of the dowel system.

When the center-to-center spacing of the dual wheels on an axle is less than 1.8*l*, the dowels located between the wheels, and some distance outside the wheels, will be stressed beyond their transfer capacity (as taken from Fig. 28) if the overlapping of stresses caused by each wheel is not taken into account. To prevent the overstressing of these dowels from this cause, the capacity factor obtained from Fig. 29 must be corrected (reduced) to compensate for the overlapping of stresses. This correction of the capacity factor may be readily accomplished by multiplying the capacity factor by the appropriate correction factor taken from Table 12. It is evident from Table 12 that even though the center-to-center spacing of the dual wheels is less than 1.8*l*, the values of the correction

Fig. 28—Load transfer capacity of single dowels that are balanced in bearing and bending.



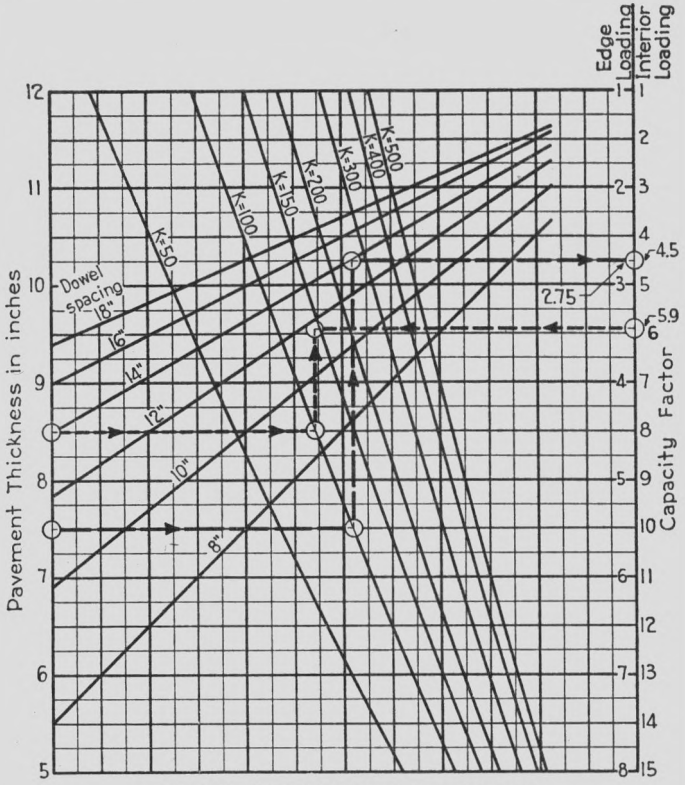


Fig. 29—Capacity factor of dowel systems.

factors do not become critical until the subgrade modulus is less than about 100 and the gross controlling wheel loads exceed about 12,000 lb.

When the center-to-center spacing of the dual wheels on an axle is less than 1.8*l* and the center of the outside dual wheel is located over the outermost dowel, the actual capacity factor of the dowels stressed by the inside dual wheel is sometimes less than the capacity factor for interior loads obtained from Fig. 29. This condition exists only in the few instances where the difference between 1.8*l* and the wheel spacing exceeds the dowel spacing. In these cases the correction (reduction) that must be made in the capacity factor is very small (maximum 2 per cent for 10-in. dowel spacing). No separate correction need be made for this condition as it was taken into account when computing the correction factors given in Table 12.

Because of the many variables involved, dowel systems are not readily designed by direct methods. However, they can be quickly designed by the "cut and try" method when Figs. 27, 28, 29 and Table 12 are used. When designing dowel systems by means of these charts and table, where applicable, the following procedure is suggested:

1. Select a dowel diameter that should prove to be adequate. For most highway loadings a dowel diameter of ¾ in. or 1 in. should be used.

Table 12

Subgrade modulus *k*	Correction factors* to be applied on capacity factors when c.-c. spacing of dual wheels is less than 1.8*l*										
	Gross controlling wheel load, lb.										
	Under 9,600	9,600	10,800	12,000	13,200	14,400	15,600	16,800	18,000	19,200	24,000
50	No correction factor required since c.-c. spacing of dual wheels equals or exceeds 1.8*l*		0.94	0.87	0.84	0.79	0.79	0.76	0.75	0.75	0.73
100					0.98	0.94	0.90	0.90	0.88	0.85	0.81
150										0.95	0.89
200											0.95
Over 200											

Note: By using values taken from Fig. 29, corrected by appropriate factor, the dowels located between the wheels will not be stressed beyond their load transfer capacity, due to the overlapping of stresses produced by each wheel.

2. Determine the length of this dowel. This may be read directly from Fig. 27 using the known or estimated joint opening*.
3. Determine the load-transfer capacity of the selected dowel when acting across the given joint opening. This may be read directly from Fig. 28.
4. Compute the required capacity factor for interior loading. This is obtained by dividing 40 to 45 per cent of the gross controlling wheel load by the load-transfer capacity of a single dowel multiplied by the appropriate correction factor taken from Table 12.
5. Determine the center-to-center spacing of the dowels that will give the required capacity factor. This spacing may be read directly from Fig. 29.

This procedure is illustrated in the following example problem:

Design a dowel system for use across the 1-in. expansion joints in a 24-ft. wide thickened-edge slab having a 10-7-10 with 2-ft. 4-in. slope cross-section, which was designed** for a 14,070-lb. gross controlling wheel load when the *k* value of the subgrade is 100.

1. A ¾-in. diameter smooth steel bar (intermediate or hard grade steel) is selected on the assumption that it will be adequate.
2. The length of this dowel that would be required when the width of joint opening is 1 in. is obtained from Fig. 27. A length of 13 in. would be used since slightly under this value is indicated.
3. The load-transfer capacity of this ¾-in. by 13-in. dowel when used across a 1-in. joint is obtained from Fig. 28. This value is 1200 lb.
4. The required capacity factor for interior loading (load on inside dual wheel when the outside dual wheel is over the outermost dowel) is computed in the following manner: from Table 12 it is found that the 14,070-lb. gross controlling wheel load and *k* of 100 require that a correction factor of 0.95 (obtained by interpolation) must be used. The required

*See page 58 for a discussion of joint openings.

**Designed in Table 7.

capacity factor for interior loading is then obtained by dividing 40 per cent of the gross controlling wheel load (14,070-lb.) by the load-transfer capacity of a single dowel (1200 lb.) multiplied by the required correction factor (0.95). Doing this, a required capacity factor of 4.9 is obtained.

$$\left(\frac{0.40\times14{,}070}{1200\times0.95}=4.9\right)$$

5. The center-to-center spacing of the dowels that will provide the required capacity (4.9) factor is read from Fig. 29. The slab has a thickened-edge cross-section so its d (8.24 in.) is used where pavement thickness is indicated. Using this (8.24), the subgrade modulus k (100), and the required capacity factor (4.9) as indicated by the short dashed line on the face of the chart, it is found that a plus 13-in. dowel spacing is needed. (Use 13 in.)

The final step in the design of a system of dowels is to check the design, obtained above, for adequacy (40 to 45 per cent load transference for interior loading and at least 20 per cent for edge loading, see page 55). The actual capacity factors for interior and edge loading that will result from using a 13-in. spacing of dowels is determined. Using Fig. 29 with an 8.24-in. d (pavement thickness), $k=100$ and the 13-in. dowel spacing, the actual capacity factors are 5.2 for interior loading and 3.1 for edge loading. These factors multiplied by the correction factor, 0.95 (previously obtained from Table 12) and the load-transfer capacity of a single dowel, 1200 lb. (previously obtained from Fig. 28), give the actual load-transfer capacity of the system for the respective type of loading. These capacities, if divided by the gross controlling wheel load, 14,070-lb., will give the percentage of load transference.

Interior loading:

$$\frac{5.2\times0.95\times1200}{14{,}070}\times100=42.1 \text{ per cent}$$

Edge loading:

$$\frac{3.1\times0.95\times1200}{14{,}070}\times100=25.1 \text{ per cent}$$

Therefore, the dowel system composed of ¾-in. diameter intermediate or hard grade smooth steel bars 13 in. in length and spaced 13 in. center-to-center will be adequate since it meets the requirements previously set forth.

It is to be noted that the results obtained by this detailed procedure are the same as those obtained in the example problem given on page 58 where the results were obtained directly from Fig. 21.

Dowel design charts, Fig. 20 for contraction joints and Fig. 21 for expansion joints, were developed so as to eliminate the foregoing lengthy design procedure. They have been designed so that when used as intended, indicated by dashed lines on their faces, they will give dowel systems that provide 40 to 45 per cent load transference for interior loading and over 20 per cent load transference for corner loadings. In short, such dowel systems will provide load transference that is adequate for Case I (protected corners) design.

APPENDIX III

A Study of Stresses in the Corner Region of Concrete Pavement Slabs Under Large Corner Loads

By Gerald Pickett*

Certain empirical formulas, which have been developed to give stresses in the corner region of concrete pavement slabs, yield unreasonable results when applied to some conditions encountered in the design of modern road and street pavements. For example, when moduli of subgrade reaction are high, yet are within the range which may be experienced in present-day pavement design, the load stresses calculated by use of these formulas sometimes decrease with decreasing thickness of pavement. This seems unreasonable and the current theoretical investigation was undertaken to clarify the situation.

The chief results of the investigation are indicated by Fig. A. Curves representing certain empirical formulas, as plotted in Fig. A, agree well throughout a portion of their length with available experimental data on stresses, but they differ in shape from curves based on theory and it appears, therefore, that they will not give accurate results throughout their entire length or even throughout as much of their length as it may be required to use in actual design problems. In fact, three of the formulas, represented by curves 1, 3 and 4 of Fig. A, indicate zero stress when the ratio $\frac{a_1}{l}$ is unity, and very low and rapidly decreasing stresses when $\frac{a_1}{l}$ is greater than about 0.6 or 0.7. The final result of the investigation is the development of a semi-empirical formula, plotted as curve 8 of Fig. A, which agrees with available experimental data as well as do the other formulas but which has the added merit of being similar in shape to curves that satisfy theoretical analyses. It is believed, therefore, that this new formula will be applicable to design over a much wider range of loads and subgrade conditions than are the other formulas.

The details of the investigation follow.

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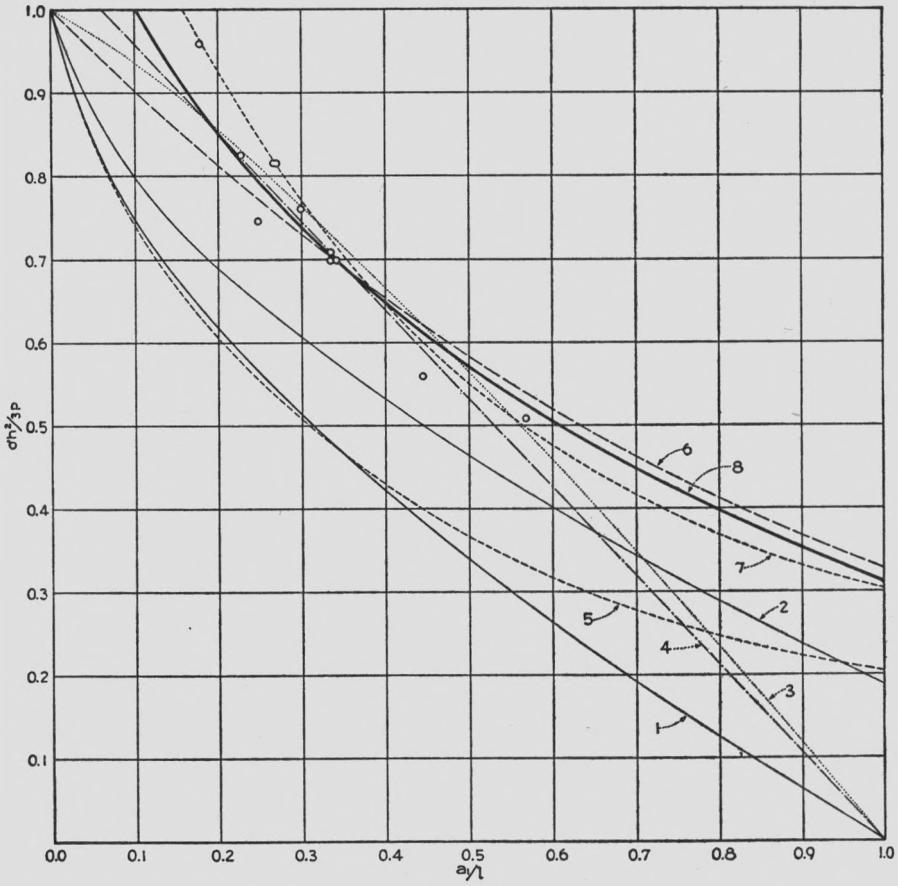
*See footnote ***, page 16.



LEGEND

Curve 1 (Westergaard)	$\frac{\sigma h^2}{3P} = 1-\left(\frac{a_1}{l}\right)^{0.6}$	Curve 5, Theoretical ———	Full subgrade support
Curve 2 (Bradbury)	$\frac{\sigma h^2}{3P} = 1-\left(\frac{a_1}{\sqrt{2}l}\right)^{0.6}$	Curve 6, Theoretical ———	Partial subgrade support
Curve 3 (Kelley)	$\frac{\sigma h^2}{3P} = 1-\left(\frac{a_1}{l}\right)^{1.2}$	Curve 7, Theoretical ———	50% increase over Curve 5
Curve 4 (Spangler)	$\frac{\sigma h^2}{3P} = \frac{3.2}{3}\left[1-\frac{a_1}{l}\right]$	Curve 8, Semi-empirical ———	$\frac{\sigma h^2}{3P} = 1.4\left[1-\frac{\sqrt{3a_1}}{1.1+\frac{0.185a_1}{l}}\right]$

° Plotted points represent experimental data furnished by the Public Roads Administration

Fig. A. Comparison of theory with various empirical and semi-empirical formulas for corner stresses in concrete pavement slabs.

As a first step in the investigation, consideration was given to several of the various empirical and semi-empirical formulas which have been developed. These included the following:

$$\sigma = \frac{3P}{h^2}\left[1 - \left(\frac{a_1}{l}\right)^{0.6}\right] \quad (1)$$

$$\sigma = \frac{3P}{h^2}\left[1 - \left(\frac{a_1}{\sqrt{2}\,l}\right)^{0.6}\right] \quad (2)$$

$$\sigma = \frac{3P}{h^2}\left[1 - \left(\frac{a_1}{l}\right)^{1.2}\right] \quad (3)$$

$$\sigma = \frac{3.2P}{h^2}\left[1 - \frac{a_1}{l}\right] \quad (4)$$

where

σ = maximum tensile stress in psi.

P = load in lb.

h = thickness of concrete slab in in.

a_1 = the distance in in. from the corner of the slab to the center of the area of load application. It is taken equal to $a\sqrt{2}$ where a is the radius of a circle equal in area to the loaded area.

$l = \left(\frac{Eh^3}{12(1-\mu^2)k}\right)^{\frac{1}{4}}$ = radius of relative stiffness in in.

E = Young's modulus for the concrete in psi.

k = subgrade modulus in psi per in.

μ = Poisson's ratio for the concrete.

It must be noted that all these equations are for Unprotected Corners. The slab corner upon which the load is imposed receives no support from adjacent slabs by means of dowels or other method of load transference.

Equation (1) was given by Dr. H. M. Westergaard, dean, Graduate School of Engineering, Harvard University [1, 2*]. It is a comparatively simple empirical equation designed to represent theoretical results with satisfactory accuracy "when a_1 and l are within plausible ranges". According to Westergaard, numerical values of maximum stresses obtained by theoretical analysis were tabulated and then compared with results obtained by equation (1).

Equation (2) was proposed by Royall D. Bradbury[3]. It is de-

*1, 2, etc., correspond to numbered references listed at end of this Appendix, page 87.

rived from equation (1) by assuming that k for the subgrade under the corner region and for some distance away from the corner is only one-fourth the normal subgrade modulus for the slab. That is, if k is reduced to $\frac{1}{4}$ its normal value, l for the corner region becomes $\sqrt{2} \times$ the normal l for the slab.

Equation (3) was proposed by E. F. Kelley[4]. It was developed by modifying the exponent in equation (1) "in such a manner as to cause the computed values to coincide more nearly with the observed data for the case of incompletely supported slab corners". The following table taken from the paper by Teller and Sutherland[5] shows the relation between the observed stresses and those computed by equation (1), for three different conditions of corner warping.

TABLE IA. Comparison of theoretical corner stresses with maximum observed stresses for three conditions of warping*.

Slab thickness, in.	Observed stresses expressed as percentages of theoretical values		
	Corner warped downward, per cent	No warping, per cent	Corner warped upward, per cent
6	114	137	144
7	116	141	149
9	105	132	138

According to Teller and Sutherland the assumed direction of corner warping was based upon the relative temperatures of the upper and lower surfaces of the slab. When the top and bottom temperatures were the same the slab was assumed to be not warped. As shown by the table all observed stresses were higher than the theoretical values and were 40 to 50 per cent higher when the corner was warped upward.

Over the range of $\frac{a_1}{l}$ represented by the conditions existing in the tests reported by Teller and Sutherland, equation (3) gives about 40 to 50 per cent higher values than equation (1) and therefore gives values in good agreement with the observed values when the corners were warped upward.

Equation (4) was proposed by M. G. Spangler[6]. It is "a simplification of Kelley's formula [equation (3)] which yields substantially

*Table 7 in reference 5.

the same results" over a considerable range of $\frac{a_1}{l}$ values. In commenting on his reason for proposing equation (4), Spangler states: "In the Arlington tests, the stress measurements which form the basis for Kelley's formula were made with the free corners of the slabs warped upward due to temperature differentials, which is the most severe case to be considered in connection with loads applied at a corner. This formula yields stresses which are compatible with the results of the experimental studies reported in this bulletin and it appears to be satisfactory as a design formula in the light of present knowledge.

"However, since this expression is purely empirical and simplification is both desirable and permissible, the fractional exponent of the quantity $\frac{a_1}{l}$ may be eliminated advantageously and the numerical coefficient of the expression raised to 3.2."

Fig. A shows a comparison of the foregoing formulas, curves 1, 2, 3 and 4 representing formulas (1), (2), (3) and (4) respectively. Also shown are three theoretical curves (curves 5, 6 and 7), one curve representing the proposed new semi-empirical formula (curve 8), and points plotted from Arlington test data and furnished by the Bureau of Public Roads.

The lower of the three theoretical curves (curve 5) is based on the assumption that the subgrade gives full support at every point under the slab. It was obtained by making the computations indicated by Westergaard[2]*. Another of the theoretical curves (curve 6) is based on the assumption that the subgrade is missing within a triangular area at the corner. The triangle is isosceles with its apex at the slab corner and its altitude equal to l, the radius of relative stiffness for the interior of the slab. The other theoretical curve (curve 7) gives stresses 50 per cent higher than curve 5; that is, a straight allowance of 50 per cent is made to account (1) for larger stresses near the bisector of the corner angle than near the edges and (2) for lack of full subgrade support.

The proposed semi-empirical formula from which curve 8 was obtained was designed to give approximately the same stresses as the Kelley and Spangler equations for the ranges of $\frac{a_1}{l}$ to which they apply and thus to be in accord with experimental data from the

*Tabulated values of such computation were used by Westergaard in selecting equation (1).

Arlington tests and also to have approximately the same shape as the theoretical curves 5, 6 and 7.

Theoretical Equations

Westergaard suggested the following equation for deflection[2]:

$$z = u_1 e^{-\frac{x}{2l}} + u_2 e^{-\frac{x}{l}} + u_3 e^{-\frac{2x}{l}} \tag{5}$$

where

z = deflection.

u_1, u_2 and u_3 = adjustable parameters.

x = a coordinate directed along the bisector of the corner.

l = the radius of relative stiffness.

The condition of minimum total potential energy, that of the slab, the subgrade and the load, gives three linear equations containing the three adjustable parameters u_1, u_2 and u_3. The equations are:

For full subgrade support:

$$\left.\begin{aligned} \frac{17}{8}u_1 + \frac{10}{9}u_2 + \frac{16}{25}u_3 &= \frac{P}{kl^2}e^{-\frac{a_1}{2l}} \\ \frac{10}{9}u_1 + u_2 + \frac{10}{9}u_3 &= \frac{P}{kl^2}e^{-\frac{a_1}{l}} \\ \frac{16}{25}u_1 + \frac{10}{9}u_2 + \frac{17}{8}u_3 &= \frac{P}{kl^2}e^{-\frac{2a_1}{l}} \end{aligned}\right\} \tag{6}$$

With support missing from $x = 0$ to $x = \beta l$:

$$\left.\begin{aligned} &\left[\frac{1}{8} + 2(1+\beta)e^{-\beta}\right]u_1 + \left[\frac{2}{9} + \frac{8}{9}\left(1+\frac{3}{2}\beta\right)e^{-\frac{3}{2}\beta}\right]u_2 \\ &\qquad + \left[\frac{8}{25} + \frac{8}{25}\left(1+\frac{5}{2}\beta\right)e^{-\frac{5}{2}\beta}\right]u_3 = \frac{P}{kl^2}e^{-\frac{a_1}{2l}} \\ &\left[\frac{2}{9} + \frac{8}{9}\left(1+\frac{3}{2}\beta\right)e^{-\frac{3}{2}\beta}\right]u_1 + \left[\frac{1}{2} + \frac{1}{2}(1+2\beta)e^{-2\beta}\right]u_2 \\ &\qquad + \left[\frac{8}{9} + \frac{2}{9}(1+3\beta)e^{-3\beta}\right]u_3 = \frac{P}{kl^2}e^{-\frac{a_1}{l}} \\ &\left[\frac{8}{25} + \frac{8}{25}\left(1+\frac{5}{2}\beta\right)e^{-\frac{5}{2}\beta}\right]u_1 + \left[\frac{8}{9} + \frac{2}{9}(1+3\beta)e^{-3\beta}\right]u_2 \\ &\qquad + \left[2 + \frac{1}{8}(1+4\beta)e^{-4\beta}\right]u_3 = \frac{P}{kl^2}e^{-\frac{2a_1}{l}} \end{aligned}\right\} \tag{7}$$

where a_1 is the distance from the corner to the center of the loaded area.

In computing the potential energy of the load it was assumed that the deflection of the center of gravity of the load was equal to the deflection of the center of the loaded area.

When the above equations are solved for u_1, u_2 and u_3 the results are as follows:

For full subgrade support:

$$\left.\begin{aligned} u_1 &= \left[2.0267\, e^{-\frac{a_1}{2l}} - 3.7555\, e^{-\frac{a_1}{l}} + 1.3533\, e^{-\frac{2a_1}{l}}\right]\frac{P}{kl^2} \\ u_2 &= \left[-3.7555\, e^{-\frac{a_1}{2l}} + 9.3455\, e^{-\frac{a_1}{l}} - 3.7555\, e^{-\frac{2a_1}{l}}\right]\frac{P}{kl^2} \\ u_3 &= \left[1.3533\, e^{-\frac{a_1}{2l}} - 3.7555\, e^{-\frac{a_1}{l}} + 2.0267\, e^{-\frac{2a_1}{l}}\right]\frac{P}{kl^2} \end{aligned}\right\} \quad (8)$$

With support missing from $x=0$ to $x=\beta l$ and β set equal to unity:

$$\left.\begin{aligned} u_1 &= \left[2.0659\, e^{-\frac{a_1}{2l}} - 4.0304\, e^{-\frac{a_1}{l}} + 1.4467\, e^{-\frac{2a_1}{l}}\right]\frac{P}{kl^2} \\ u_2 &= \left[-4.0304\, e^{-\frac{a_1}{2l}} + 11.5656\, e^{-\frac{a_1}{l}} - 4.5400\, e^{-\frac{2a_1}{l}}\right]\frac{P}{kl^2} \\ u_3 &= \left[1.4467\, e^{-\frac{a_1}{2l}} - 4.5400\, e^{-\frac{a_1}{l}} + 2.3071\, e^{-\frac{2a_1}{l}}\right]\frac{P}{kl^2} \end{aligned}\right\} \quad (9)$$

The moment at a section that is perpendicular to the bisector of the corner angle and distant x from the corner for $x > a_1$, was determined by the formula.

$$M = P\,(x-a_1) - \int_0^x kz\,(x-v)(2v\,dv) \qquad (10)$$

where k was taken to be zero for regions in which the subgrade was assumed to be missing.

Equation (10) can be put into the following form:

$$\frac{M}{Pl} = \left(\frac{x}{l} - \frac{a_1}{l}\right) - \frac{kl^2}{P}\left[u_1 I_1 + u_2 I_2 + u_3 I_3\right] \qquad (11)$$

From equations (8), (9), (10) and (11) the following equations for the factors I_1, I_2 and I_3 are obtained:

For full subgrade support:

$$\left.\begin{aligned} I_1 &= 8\left[-4+\frac{x}{l}+\left(4+\frac{x}{l}\right)e^{-\frac{x}{2l}}\right] \\ I_2 &= 2\left[-2+\frac{x}{l}+\left(2+\frac{x}{l}\right)e^{-\frac{x}{l}}\right] \\ I_3 &= \frac{1}{2}\left[-1+\frac{x}{l}+\left(1+\frac{x}{l}\right)e^{-\frac{2x}{l}}\right] \end{aligned}\right\} \qquad (12)$$

With support missing from $x=0$ to $x=l$:

$$\left.\begin{aligned} I_1 &= 12\left(-\frac{13}{3}+\frac{x}{l}\right)e^{-\frac{1}{2}}+8\left(4+\frac{x}{l}\right)e^{-\frac{x}{2l}} \\ I_2 &= 4\left(-\frac{5}{2}+\frac{x}{l}\right)e^{-1}+2\left(2+\frac{x}{l}\right)e^{-\frac{x}{l}} \\ I_3 &= \frac{3}{2}\left(-\frac{5}{3}+\frac{x}{l}\right)e^{-2}+\frac{1}{2}\left(1+\frac{x}{l}\right)e^{-\frac{2x}{l}} \end{aligned}\right\} \qquad (13)$$

On the assumption that the moment is uniformly distributed along the section under consideration (section perpendicular to the bisector of the corner angle) the flexure formula gives the following equation for stress in the outer fiber:

$$\sigma=\frac{3M}{h^2x} \qquad (14)$$

From equations (11) and (14):

$$\frac{\sigma h^2}{3P}=\frac{l}{x}\left[\left(\frac{x}{l}-\frac{a_1}{l}\right)-\frac{kl^2}{P}\left(u_1I_1+u_2I_2+u_3I_3\right)\right] \qquad (15)$$

The $\frac{x}{l}$ that would give a maximum value of $\frac{\sigma h^2}{3P}$ for selected values of $\frac{a_1}{l}$ and for each assumption in regard to subgrade was obtained by trial and the corresponding values of $\frac{\sigma h^2}{3P}$ used for plotting curves 5 and 6 for the two assumptions in regard to subgrade.

Proposed Semi-empirical Equation

It is readily apparent from Fig. A that none of the empirical equations previously used are applicable for relatively high values of $\frac{a_1}{l}$. The Westergaard, Kelley and Spangler formulas all give zero stress for $\frac{a_1}{l}$ equal to unity. For an empirical equation to be appli-

cable over a comparatively wide range it should first be in accord with experimental data within the range for which reliable data are available, and second, when represented graphically it should give curves similar to those obtained from theoretical analysis.

The extent to which extrapolation of an empirical equation may be carried safely will depend upon how well the reasons for the discrepancy between the available theory and experiment are understood and what use is made of this knowledge in formulating the empirical equation. On the basis of the experiments at Arlington, Kelley and his associates conclude that lack of subgrade support in the corner region is the primary cause of experimental stresses being larger than theoretical stresses[4, 5]. On the basis of similar data supplemented by mathematical treatment Spangler believes that a deviation in distribution of moments from that assumed in the theoretical development is a major factor[6]. Under the heading "A New Hypothesis of Stress Distribution in the Corner Region" (*reference 6, pages 27 to 31*), Spangler reasons that this factor alone could account for actual maximum stresses being 53 per cent higher than the theoretical stresses.

If Spangler's analysis is valid then the empirical curve should approximately coincide with curve 7 of Fig. A. However if lack of subgrade support is the cause of the higher experimentally obtained stresses, the empirical curve should be similar in shape to curve 6 or some similar curve based upon lack of subgrade support.

Spangler in his analysis assumed that along a circular arc with the corner as a center, the moment decreases as the edges are approached and that the direction of the moment is normal to the arc*. His data and those of the Arlington tests indicate a decrease in moment near the edges but the direction of the moment is more nearly parallel to the bisector of the corner angle than it is normal to the circular arc. Fig. 20 of reference 6 shows the direction of principal stresses to be nearly parallel to the bisector; therefore the direction of principal moments would be in approximately the same direction. If, in Spangler's analysis, the principal moments had been considered to be parallel to the bisector the result would have been

*As pointed out by Spangler, this is the reverse of findings obtained by Murphy whose analytical solution yielded greater stresses at the edge than along the bisector[7]. However, Murphy used only a limited number of terms of a relatively slowly converging double-infinite series. It is believed that the use of a sufficient number of terms would have shown Murphy's theoretical solution to be much more in accord with the experimental results of Spangler.

an increase of 29 per cent over the theoretical stresses instead of 53 per cent increase. An intermediate value may be more nearly correct.

In choosing the semiempirical equation

$$\sigma=\frac{4.2\,P}{h^2}\left[1-\frac{\sqrt{\frac{a_1}{l}}}{1.1+\frac{0.185\,a_1}{l}}\right]=\frac{4.2\,P}{h^2}\left[1-\frac{\sqrt{\frac{a}{l}}}{0.925+0.22\frac{a}{l}}\right] \quad (16)$$

from which curve 8 was constructed, both factors discussed above were considered. A 40 per cent increase in stress was allowed due to nonuniform distribution of moment along sections perpendicular to the bisector of the corner angle. Further allowance was made for lack of subgrade support. The formula was selected to give stresses within 3 per cent of those given by either the Kelley or Spangler formulas in the range of $\frac{a_1}{l}$ between 0.1 and 0.4, and therefore to be in good agreement with the Arlington experimental data. The formula should be fairly reliable for $\frac{a_1}{l}$ as large as 1.0 which is probably well beyond the range of values that will be encountered in any current pavement design problem. Some modification might be desirable for $\frac{a_1}{l}$ much larger than 1.0.

Like the other equations on page 79, this equation is for an unprotected corner, a free corner which receives no support from adjacent slabs.

The question might arise as to why a semiempirical equation is proposed when more reliable results could be obtained theoretically by a theory that is properly modified to take deficient subgrade support and other variables into account. The answer is that the theory leads to formulas involving tedious computations, whereas a comparatively simple empirical formula can be selected which will give practically the same results over any arbitrarily chosen range.

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